Theorem **Rational Root Theorem**

If $\frac{p}{q}$ is in simplest form and is a rational root of the polynomial equation $a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0 = 0$ with integer coefficients, then p must be a factor of a_0 and q must be a factor of a_n.

OBJECTIVE 1

1 EXAMPLE Find the rational roots of $3x^3 - x^2 - 15x + 5$.

Step 1: List the possible rational roots.

The leading coefficient is 3. The constant term is 5. By the Rational Root Theorem, *the only possible rational roots of the equation have the form* $\frac{\text{factors of 5}}{\text{factors of 3}}$.

The factors of 5 are ±5 and ±1 and ±5. The factors of 3 are ±3 and ±1. The only possible rational roots are $\pm 5, \pm\frac{5}{3}, \pm 1, \pm\frac{1}{3}$.

OBJECTIVE 1

EXAMPLE 1 **(continued)**

Step 2: Test each possible rational root.

$5: 3(5)^3 - (5)^2 - 15(5) + 5 = 280 \neq 0$

$-5: 3(-5)^3 - (-5)^2 - 15(-5) + 5 = -320 \neq 0$

$\frac{5}{3}: 3\left(\frac{5}{3}\right)^3 - \left(\frac{5}{3}\right)^2 - 15\left(\frac{5}{3}\right) + 5 = -8.\overline{8} \neq 0$

$-\frac{5}{3}: 3\left(-\frac{5}{3}\right)^3 - \left(-\frac{5}{3}\right)^2 - 15\left(-\frac{5}{3}\right) + 5 = -13.\overline{3} \neq 0$

$1: 3(1)^3 - (1)^2 - 15(1) + 5 = -8 \neq 0$

$-1: 3(-1)^3 - (-1)^2 - 15(-1) + 5 = 16 \neq 0$

$\frac{1}{3}: 3\left(\frac{1}{3}\right)^3 - \left(\frac{1}{3}\right)^2 - 15\left(\frac{1}{3}\right) + 5 = 0$ So $\frac{1}{3}$ is a root.

$-\frac{1}{3}: 3\left(-\frac{1}{3}\right)^3 - \left(-\frac{1}{3}\right)^2 - 15\left(-\frac{1}{3}\right) + 5 = 9.\overline{7} \neq 0$

The only rational root of $3x^3 - x^2 - 15x + 5 = 0$ is $\frac{1}{3}$.

OBJECTIVE 1

2 EXAMPLE Find the roots of $5x^3 - 24x^2 + 41x - 20 = 0$.

Step 1: List the possible rational roots.

The leading coefficient is 5. The constant term is 20. By the Rational Root Theorem, *the only possible roots of the equation have the form* $\frac{\text{factors of } -20}{\text{factors of } 5}$.

The factors of –20 are ±1 and ±20, ±2 and ±10, and ±4 and ±5. The only factors of 5 are ±1 and ±5. The only possible rational roots are $\pm\frac{1}{5}$, $\pm\frac{2}{5}$, $\pm\frac{4}{5}$, ±1, ±2, ±4, ±5, ±10, and ±20.

OBJECTIVE 1

2 EXAMPLE **(continued)**

Step 2: Test each possible rational root until you find a root.

Test $\frac{1}{5}$: $5\left(\frac{1}{5}\right)^3 - 24\left(\frac{1}{5}\right)^2 \pm 41\left(\frac{1}{5}\right) - 20 = -12.72 \neq 0$

Test $-\frac{1}{5}$: $5\left(-\frac{1}{5}\right)^3 - 24\left(-\frac{1}{5}\right)^2 \pm 41\left(-\frac{1}{5}\right) - 2 = -29.2 \neq 0$

Test $\frac{2}{5}$: $5\left(\frac{2}{5}\right)^3 - 24\left(\frac{2}{5}\right)^2 \pm 41\left(\frac{2}{5}\right) - 20 = -7.12 \neq 0$

Test $-\frac{2}{5}$: $5\left(-\frac{2}{5}\right)^3 - 24\left(-\frac{2}{5}\right)^2 \pm 41\left(-\frac{2}{5}\right) - 20 = -40.56 \neq 0$

Test $\frac{4}{5}$: $5\left(\frac{4}{5}\right)^3 - 24\left(\frac{4}{5}\right)^2 \pm 41\left(\frac{4}{5}\right) - 20 = 0$ So $\frac{4}{5}$ is a root.

Step 3: Use synthetic division with the root you found in Step 2 to find the quotient.

$\frac{4}{5}$	5	–24	41	–20
		4	–16	20
	5	–20	25	0
	↓	↓	↓	↓
	$5x^2$	$-\ 20x$	$+\ 25$	Remainder

OBJECTIVE 1

2 EXAMPLE **(continued)**

Step 4: Find the roots of $5x^2 - 20x + 25 = 0$.

$5x^2 - 20x + 25 = 0$	
$5(x^2 - 4x + 5) = 0$	Factor out the GCF, 5.
$x^2 - 4x + 5 = 0$	
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Quadratic Formula
$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)}$	Substitute 1 for *a*, –4 for *b*, and 5 for *c*.
$= \frac{4 \pm \sqrt{-4}}{2}$	Use order of operations.
$= \frac{4 \pm 2i}{2}$	$\sqrt{-1} = i$.
$= 2 \pm i$	Simplify.

The roots of $5x^3 - 24x^2 + 41x - 20 = 0$ are $\frac{4}{5}$, $2 + i$, and $2 - i$.