Objective

The student will be able to:

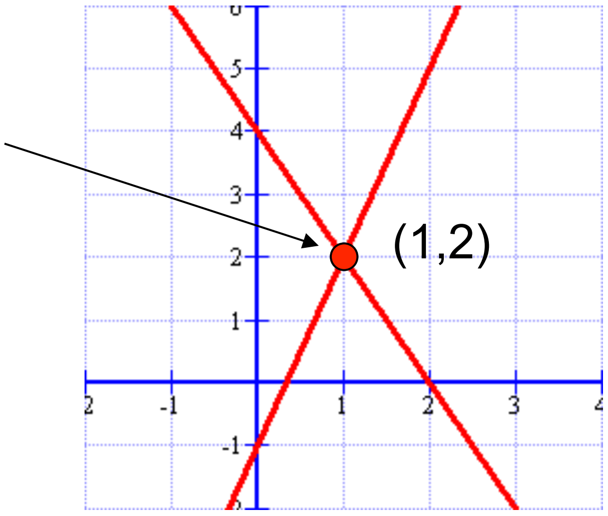
solve systems of equations by graphing.

What is a system of equations?

- A system of equations is when you have two or more equations using the same variables.
- The solution to the system is the point that satisfies ALL of the equations. This point will be an ordered pair.
- When graphing, you will encounter three possibilities.

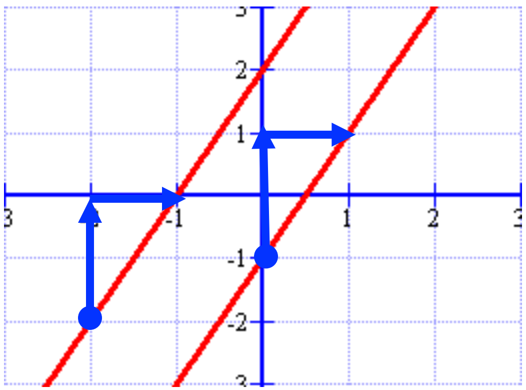
Intersecting Lines

- The point where the lines intersect is your solution.
- The solution of this graph is (1, 2)



Parallel Lines

- These lines never intersect!
- Since the lines never cross, there is **NO SOLUTION**!
- Parallel lines have the same slope with different y-intercepts.



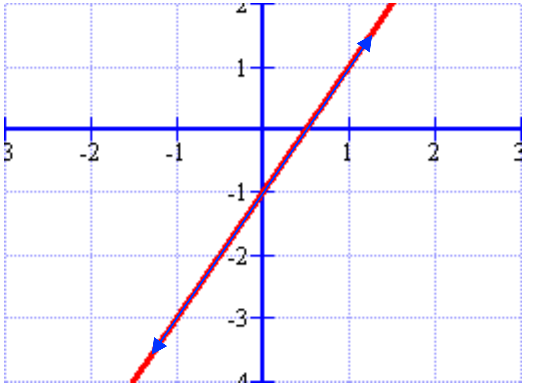
$$\text{Slope} = \frac{2}{1} = 2$$

y-intercept = 2

y-intercept = -1

Coinciding Lines

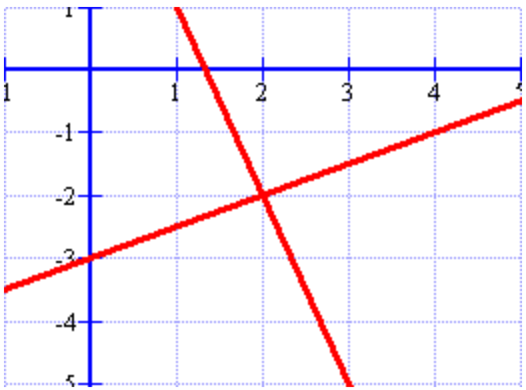
- These lines are the same!
- Since the lines are on top of each other, there are **INFINITELY MANY SOLUTIONS!**
- Coinciding lines have the same slope and y-intercepts.



$$\text{Slope} = \frac{2}{1} = 2$$

$$\text{y-intercept} = -1$$

What is the solution of the system graphed below?



1. (2, -2)
2. (-2, 2)
3. No solution
4. Infinitely many solutions

1) Find the solution to the following system:

$$2x + y = 4$$
$$x - y = 2$$

Graph both equations. I will graph using x- and y-intercepts (plug in zeros).

$2x + y = 4$	$x – y = 2$
(0, 4) and (2, 0)	(0, -2) and (2, 0)

Graph the ordered pairs.

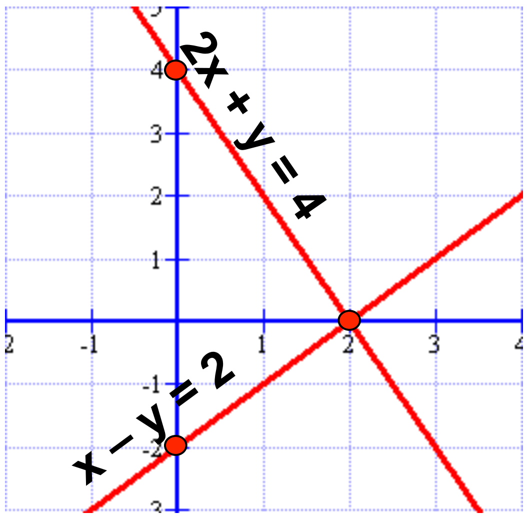
Graph the equations.

2x + y = 4

(0, 4) and (2, 0)

x - y = 2

(0, -2) and (2, 0)

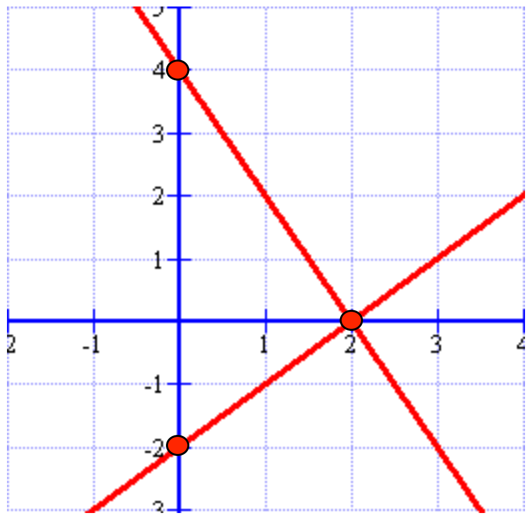


Where do the lines intersect?

(2, 0)

Check your answer!

To check your answer, plug the point back into both equations.



$2x + y = 4$

$2(2) + (0) = 4$ ✓

$x - y = 2$

$(2) – (0) = 2$ ✓

Nice job…let’s try another!

2) Find the solution to the following system:

$y = 2x – 3$

$-2x + y = 1$

Graph both equations. Put both equations in slope-intercept or standard form. I'll do slope-intercept form on this one!

$y = 2x – 3$

$y = 2x + 1$

Graph using slope and y-intercept

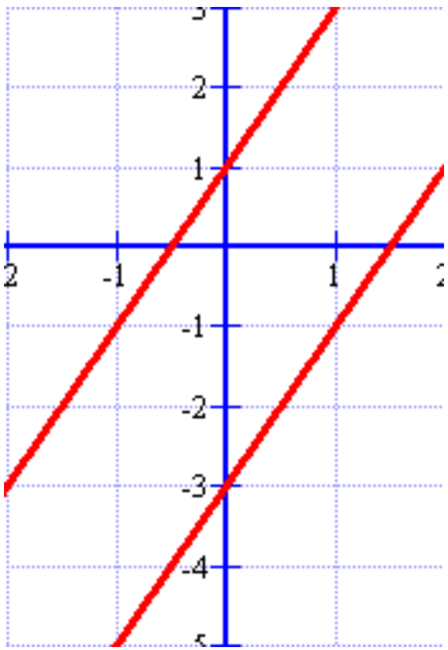
Graph the equations.

$y = 2x - 3$

$m = 2$ and $b = -3$

$y = 2x + 1$

$m = 2$ and $b = 1$



Where do the lines intersect?

No solution!

Notice that the slopes are the same with different y-intercepts. If you recognize this early, you don't have to graph them!

What is the solution of this system?

$3x - y = 8$
$2y = 6x - 16$

1. (3, 1)
2. (4, 4)
3. No solution
4. ✓ Infinitely many solutions

Solving a system of equations by graphing.

Let's summarize! There are **3 steps** to solving a system using a graph.

Step 1: Graph both equations.

Graph using slope and y – intercept or x- and y-intercepts. Be sure to use a ruler and graph paper!

Step 2: Do the graphs intersect?

This is the solution! LABEL the solution!

Step 3: Check your solution.

Substitute the x and y values into both equations to verify the point is a solution to both equations.

Objective

The student will be able to:

solve systems of equations using substitution.

Solving Systems of Equations

- You can solve a system of equations using different methods. The idea is to determine which method is easiest for that particular problem.
- These notes show how to solve the system algebraically using SUBSTITUTION.

Solving a system of equations by substitution

Step 1: Solve an equation for one variable.	Pick the easier equation. The goal is to get y= ; x= ; a= ; etc.
Step 2: Substitute	Put the equation solved in Step 1 into the other equation.
Step 3: Solve the equation.	Get the variable by itself.
Step 4: Plug back in to find the other variable.	Substitute the value of the variable into the equation.
Step 5: Check your solution.	Substitute your ordered pair into BOTH equations.

1) Solve the system using substitution

$$x + y = 5$$

$$y = 3 + x$$

Step 1: Solve an equation for one variable.	The second equation is already solved for y!
Step 2: Substitute	$x + y = 5$ $x + (\mathbf{3 + x}) = 5$
Step 3: Solve the equation.	$2x + 3 = 5$ $2x = 2$ $x = 1$

1) Solve the system using substitution

$$x + y = 5$$
$$y = 3 + x$$

Step 4: Plug back in to find the other variable.

$$x + y = 5$$
$$(1) + y = 5$$
$$y = 4$$

Step 5: Check your solution.

(1, 4)

$(1) + (4) = 5$ ✓

$(4) = 3 + (1)$ ✓

The solution is (1, 4). What do you think the answer would be if you graphed the two equations?

Which answer checks correctly?

$$3x - y = 4$$
$$x = 4y - 17$$

1. (2, 2)
2. (5, 3)
3. ✓ (3, 5)
4. (3, -5)

2) Solve the system using substitution

$$3y + x = 7$$

$$4x - 2y = 0$$

Step 1: Solve an equation for one variable.

It is easiest to solve the first equation for x.

$$3y + x = 7$$

$$\underline{-3y \qquad -3y}$$

$$x = -3y + 7$$

Step 2: Substitute

$$4x - 2y = 0$$

$$4(-3y + 7) - 2y = 0$$

2) Solve the system using substitution

$$3y + x = 7$$

$$4x - 2y = 0$$

Step 3: **Solve the equation.**

$$-12y + 28 - 2y = 0$$
$$-14y + 28 = 0$$
$$-14y = -28$$
$$\mathbf{y = 2}$$

Step 4: **Plug back in to find the other variable.**

$$4x - 2y = 0$$
$$4x - 2(\mathbf{2}) = 0$$
$$4x - 4 = 0$$
$$4x = 4$$
$$\mathbf{x = 1}$$

2) Solve the system using substitution

$$3y + x = 7$$

$$4x - 2y = 0$$

Step 5: Check your solution.

(1, 2)

$3(2) + (1) = 7$ ✓

$4(1) - 2(2) = 0$ ✓

When is solving systems by substitution easier to do than graphing?

When only one of the equations has a variable already isolated (like in example #1).

If you solved the first equation for x, what would be substituted into the bottom equation.

$$2x + 4y = 4$$
$$3x + 2y = 22$$

1. $-4y + 4$
2. ✓ $-2y + 2$
3. $-2x + 4$
4. $-2y + 22$

3) Solve the system using substitution

$x = 3 - y$

$x + y = 7$

Step 1: Solve an equation for one variable.	The first equation is already solved for x!
Step 2: Substitute	$x + y = 7$ $(\mathbf{3 - y}) + y = 7$
Step 3: Solve the equation.	$3 = 7$ The variables were eliminated!! This is a special case. Does 3 = 7? FALSE!

When the result is FALSE, the answer is NO SOLUTIONS.

3) Solve the system using substitution

$$2x + y = 4$$
$$4x + 2y = 8$$

Step 1: Solve an equation for one variable.

The first equation is easiest to solved for y!

$y = -2x + 4$

Step 2: Substitute

$4x + 2y = 8$

$4x + 2(-2x + 4) = 8$

Step 3: Solve the equation.

$4x - 4x + 8 = 8$

$8 = 8$

This is also a special case.

Does 8 = 8? TRUE!

When the result is TRUE, the answer is **INFINITELY MANY SOLUTIONS**.

What does it mean if the result is "TRUE"?

1. The lines intersect
2. The lines are parallel
3. ✓ The lines are coinciding
4. The lines reciprocate
5. I can spell my name

Objective

The student will be able to:

solve systems of equations using elimination with multiplication.

Solving a system of equations by elimination using multiplication.

Step 1: Put the equations in Standard Form.	Standard Form: Ax + By = C
Step 2: Determine which variable to eliminate.	Look for variables that have the same coefficient.
Step 3: Multiply the equations and solve.	Solve for the variable.
Step 4: Plug back in to find the other variable.	Substitute the value of the variable into the equation.
Step 5: Check your solution.	Substitute your ordered pair into BOTH equations.

1) Solve the system using elimination.

$$2x + 2y = 6$$

$$3x - y = 5$$

Step 1: **Put the equations in Standard Form.**

They already are!

Step 2: **Determine which variable to eliminate.**

None of the coefficients are the same!

Find the least common multiple of each variable.

LCM = 6x, LCM = 2y

Which is easier to obtain?

2y

(you only have to multiply the bottom equation by 2)

1) Solve the system using elimination.

$$2x + 2y = 6$$
$$3x - y = 5$$

Step 3: Multiply the equations and solve.

Multiply the bottom equation by 2

$2x + 2y = 6$

$(2)(3x - y = 5)$

$2x + 2y = 6$

$(+) \quad 6x - 2y = 10$

$8x = 16$

$x = 2$

Step 4: Plug back in to find the other variable.

$2(2) + 2y = 6$

$4 + 2y = 6$

$2y = 2$

$y = 1$

1) Solve the system using elimination.

$$2x + 2y = 6$$
$$3x - y = 5$$

Step 5: Check your solution.

(2, 1)

2(2) + 2(1) = 6 ✓

3(2) - (1) = 5 ✓

Solving with multiplication adds one more step to the elimination process.

2) Solve the system using elimination.

$$x + 4y = 7$$

$$4x - 3y = 9$$

Step 1: Put the equations in Standard Form.

They already are!

Step 2: Determine which variable to eliminate.

Find the least common multiple of each variable.

LCM = 4x, LCM = 12y

Which is easier to obtain?

4x

(you only have to multiply the top equation by -4 to make them inverses)

2) Solve the system using elimination.

$$x + 4y = 7$$
$$4x - 3y = 9$$

Step 3: Multiply the equations and solve.

Multiply the top equation by -4

$(-4)(x + 4y = 7) \quad -4x - 16y = -28$

$4x - 3y = 9) \quad (+)\ 4x - 3y = 9$

$-19y = -19$

y = 1

Step 4: Plug back in to find the other variable.

$x + 4(1) = 7$

$x + 4 = 7$

x = 3

2) Solve the system using elimination.

$$x + 4y = 7$$

$$4x – 3y = 9$$

Step 5: Check your solution.

(3, 1)

(3) + 4(1) = 7 ✓

4(3) - 3(1) = 9 ✓

What is the first step when solving with elimination?

1. Add or subtract the equations.
2. Multiply the equations.
3. Plug numbers into the equation.
4. Solve for a variable.
5. Check your answer.
6. Determine which variable to eliminate.
7. Put the equations in standard form.

Which variable is easier to eliminate?

$$3x + y = 4$$
$$4x + 4y = 6$$

1. x
2. ✓ y
3. 6
4. 4

3) Solve the system using elimination.

$$3x + 4y = -1$$

$$4x - 3y = 7$$

Step 1: Put the equations in Standard Form.

They already are!

Step 2: Determine which variable to eliminate.

Find the least common multiple of each variable.

LCM = 12x, LCM = 12y

Which is easier to obtain?

Either! I'll pick y because the signs are already opposite.

3) Solve the system using elimination.

$$3x + 4y = -1$$

$$4x - 3y = 7$$

Step 3: Multiply the equations and solve.

Multiply both equations

$(3)(3x + 4y = -1)$ → $9x + 12y = -3$

$(4)(4x - 3y = 7)$ → $(+)\ 16x - 12y = 28$

$25x = 25$

$x = 1$

Step 4: Plug back in to find the other variable.

$3(1) + 4y = -1$

$3 + 4y = -1$

$4y = -4$

$y = -1$

3) Solve the system using elimination.

$$3x + 4y = -1$$

$$4x - 3y = 7$$

Step 5: Check your solution.

(1, -1)

$3(1) + 4(-1) = -1$ ✓

$4(1) - 3(-1) = 7$ ✓

What is the best number to multiply the top equation by to eliminate the x’ s?

$$3x + y = 4$$
$$6x + 4y = 6$$

1. -4
2. ✓ -2
3. 2
4. 4

Solve using elimination.

$$2x - 3y = 1$$
$$x + 2y = -3$$

1. (2, 1)
2. (1, -2)
3. (5, 3)
4. (-1, -1) ✓