4–1 Introduction

Many problems in probability and statistics require a careful analysis of the outcomes of a sequence of events. A sequence of events occurs when one or more events follow one another. For example, a sales representative may wish to select the most efficient way to visit several different stores in four cities.

Sometimes, in order to determine costs or rates, a manager must know all possible outcomes of a classification scheme. Or an insurance company may wish to classify its drivers according to the following classes:

1. Gender (male, female).
2. Age (under 25, between 25 and 60, over 60).
3. Area of residence (rural, suburban, urban).
4. Distance driven to work (under 4 miles, between 4 and 10 miles, over 10 miles).
5. Value of the vehicle (under $5000, between $5000 and $10,000, over $10,000).

We can think of falling into one of these classes or categories as an "event." Then the particular set of categories (female, under 25, suburban, etc.) that a person is in is like a sequence of events.

In a psychological study, a researcher might attempt to train a rat to run a maze. To determine the rat's success, the researcher must know the number of possible choices the rat can make to traverse the maze. Then the researcher can differentiate between actual learning and chance successes.

On a game show, a contestant might be required to arrange five digits correctly to guess the exact price of a new car. To determine the probability of his guessing the correct answer, one must know the number of possible ways the five digits can be arranged.

The vice president of a company might wish to know the number of different possible ways four employees can be selected from a group of 10 in order to be transferred to a new location.

Sometimes the total number of possible outcomes is enough; other times a list of all outcomes is needed. One can use several methods of counting here: the multiplication rule, the permutation rule, and the combination rule.

4–2 Tree Diagrams and the Multiplication Rule for Counting

Tree Diagrams

Objective 1. Determine the number of outcomes of a sequence of events using a tree diagram.

Many times one wishes to list each possibility of a sequence of events. For example, it would be difficult to list all possible outcomes of the options available on a new automobile by guessing alone. Rather than do this listing in a haphazard way, one can use a tree diagram.

A **tree diagram** is a device used to list all possibilities of a sequence of events in a systematic way.

Tree diagrams are also useful in determining the probabilities of events, as will be shown in the next chapter.

Example 4–1

Suppose a sales rep can travel from New York to Pittsburgh by plane, train, or bus, and from Pittsburgh to Cincinnati by bus, boat, or automobile. List all possible ways he can travel from New York to Cincinnati.

Solution

A tree diagram can be drawn to show the possible ways. First, the salesman can travel from New York to Pittsburgh by three methods. The tree diagram for this situation is shown in Figure 4–1.

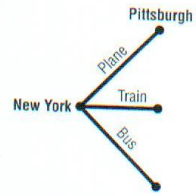

Figure 4–1
Tree Diagram for New York–Pittsburgh Trips in Example 4–1

Then the salesman can travel from Pittsburgh to Cincinnati by bus, boat, or automobile. This tree diagram is shown in Figure 4–2.

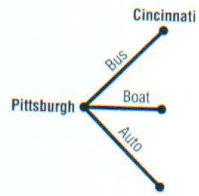

Figure 4–2
Tree Diagram for Pittsburgh–Cincinnati Trips in Example 4–1

Next, the second branch is paired up with the first branch in three ways, as shown in Figure 4–3.

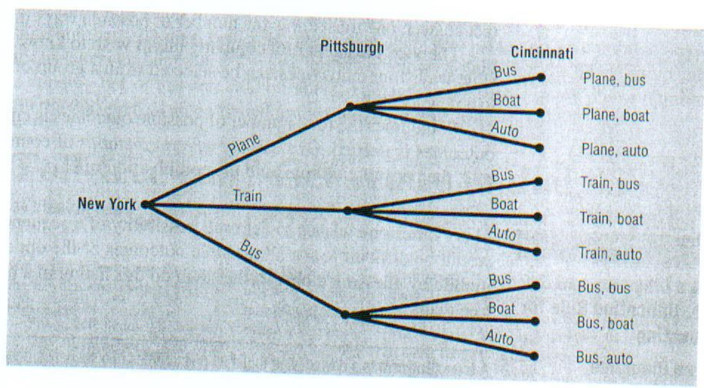

Figure 4–3
Complete Tree Diagram for Example 4–1

Finally, all outcomes can be listed by starting at New York and following the branches to Cincinnati, as shown at the right end of the tree in Figure 4–3. There are nine different ways.

Example 4–2

A coin is tossed and a die is rolled. Find all possible outcomes of this sequence of events.

Solution

Since the coin can land either heads up or tails up, and since the die can land with any one of six numbers shown face up, the outcomes can be represented as shown in Figure 4–4.

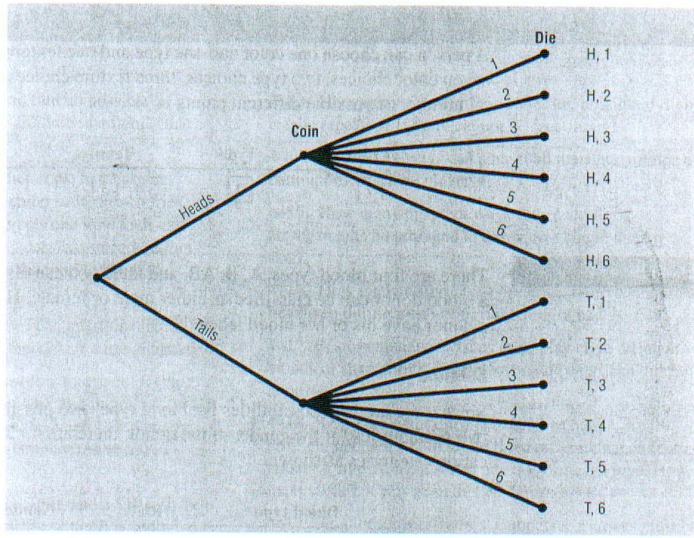

Figure 4–4
Complete Tree Diagram for Example 4–2

Interesting Facts

Possible games of chess: 25×10^{115}. (*The Harper's Index Book*, p. 36)

The Multiplication Rule for Counting

In order to determine the total number of outcomes in a sequence of events, the *multiplication* rule can be used.

Objective 2. Find the total number of outcomes in a sequence of events using the multiplication rule.

Multiplication Rule

In a sequence of n events in which the first one has k_1 possibilities and the second event has k_2 and the third has k_3, and so forth, the total number of possibilities of the sequence will be

$$k_1 \cdot k_2 \cdot k_3 \cdot \cdots \cdot k_n$$

Note: "And" in this case means to multiply.

The next examples illustrate the multiplication rule.

Example 4–3

A paint manufacturer wishes to manufacture several different paints. The categories include

Color	Red, blue, white, black, green, brown, yellow
Type	Latex, oil
Texture	Flat, semigloss, high gloss
Use	Outdoor, indoor

How many different kinds of paint can be made if a person can select one color, one type, one texture, and one use?

Solution

A person can choose one color and one type and one texture and one use. Since there are seven color choices, two type choices, three texture choices, and two use choices, the total number of possible different paints is

Color		Type		Texture		Use	
7	·	2	·	3	·	2	= 84

Example 4–4

There are four blood types, A, B, AB, and O. Blood can also be Rh+ and Rh−. Finally, a blood donor can be classified as either male or female. How many different ways can a donor have his or her blood labeled?

Solution

Since there are four possibilities for blood type, two possibilities for the Rh factor, and two possibilities for the gender of the donor, there are $4 \cdot 2 \cdot 2$, or 16, different classification categories as shown.

Blood type		Rh		Gender	
4	·	2	·	2	= 16

When determining the number of different possibilities of a sequence of events, one must know whether repetitions are permissible.

Example 4–5

The digits 0, 1, 2, 3, and 4 are to be used in a four-digit ID card. How many different cards are possible if repetitions are permitted?

Solution

Since there are four spaces to fill and five choices for each space, the solution is

$$5 \cdot 5 \cdot 5 \cdot 5 = 5^4 = 625$$

Now, what if repetitions are not permitted? For Example 4–5, the first digit can be chosen in five ways. But the second digit can be chosen in only four ways, since there are only four digits left; etc. Thus, the solution is

$$5 \cdot 4 \cdot 3 \cdot 2 = 120$$

The same situation occurs when one is drawing balls from an urn or cards from a deck. If the ball or card is replaced before the next one is selected, then repetitions are permitted, since the same one can be selected again. But if the selected ball or card is not replaced, then repetitions are not permitted, since the same ball or card cannot be selected the second time.

These examples illustrate the multiplication rule. In summary: if repetitions are permitted, then the numbers stay the same going from left to right. If repetitions are "not" permitted, then the numbers decrease by one for each place left to right.

Exercises

4–1. By means of a tree diagram, find all possible outcomes for the genders of the children in a family that has three children.

4–2. Bill's Burger Palace sells hot dogs, hamburgers, cheeseburgers, root beer, cola, lemon soda, french fries, and baked potatoes. If a customer selects one sandwich, one drink, and one potato, how many possible selections can the customer make? Draw a tree diagram to show the possibilities.

4–3. A quiz consists of four true–false questions. How many possible answer keys are there? Use a tree diagram.

4–4. Students are classified according to eye color (blue, brown, green), gender (male, female), and major (chemistry, mathematics, physics, business). How many possible different classifications are there? Use a tree diagram.

4–5. A box contains a $1 bill, a $5 bill, and a $10 bill. Two bills are selected in succession, without the first bill being replaced. Draw a tree diagram and represent all possible amounts of money that can be selected.

4–6. The Eagles and the Hawks play three games of hockey. Draw a tree diagram to represent the outcomes of the victories.

4–7. An inspector selects three batteries from a lot, then tests each to see whether each is overcharged, normal, or undercharged. Draw a tree diagram to represent all possible outcomes.

4–8. Draw a tree diagram to represent the outcomes when two players flip coins to see whether or not they match.

4–9. A coin is tossed. If it comes up heads, it is tossed again. If it lands tails, a die is rolled. Find all possible outcomes of this sequence of events.

4–10. A person has a chance of obtaining a degree from each category listed below. Draw a tree diagram showing all possible ways a person could obtain these degrees.

Bachelor's	Master's	Doctor's
B.S.	M.S.	Ph.D.
B.A.	M.Ed.	D.Ed.
	M.A.	

4–11. If blood types can be A, B, AB, and O, and Rh+ and Rh−, draw a tree diagram for the possibilities.

4–12. A woman has three skirts, five blouses, and four scarves. How many different outfits can she wear, assuming that they are color-coordinated?

4–13. How many five-digit zip codes are possible if digits can be repeated? If there cannot be repetitions?

4–14. How many ways can a baseball manager arrange a batting order of nine players?

4–15. How many different ways can seven floral arrangements be arranged in a row on a single display shelf?

4–16. How many different ways can six radio commercials be played during a one-hour radio program?

4–17. A store manager wishes to display eight different brands of shampoo in a row. How many ways can this be done?

4–18. There are eight different statistics books, six different geometry books, and three different trigonometry books. A student must select one book of each type. How many different ways can this be done?

4–19. At a local cheerleaders' camp, five routines must be practiced. A routine may not be repeated. In how many different orders can these five routines be presented?

4–20. The call letters of a radio station must have four letters. The first letter must be a *K* or a *W*. How many different station call letters can be made if repetitions are not allowed? If repetitions are allowed?

4–21. How many different three-digit identification tags can be made if the digits can be used more than once? If the first digit must be a 5 and repetitions are not permitted?

4–22. How many different ways can nine trophies be arranged on a shelf?

4–23. If a baseball manager has five pitchers and two catchers, how many different possible pitcher–catcher combinations can he field?

4–24. There are two major roads from city *X* to city *Y*, and four major roads from city *Y* to city *Z*. How many different trips can be made from city *X* to city *Z* passing through city *Y*?

***4–25.** Pine Pizza Palace sells pizza plain or with one or more of the following toppings: pepperoni, sausage, mushrooms, olives, onions, or anchovies. How many different pizzas can be made? (*Hint:* A person can select or not select each item.)

***4–26.** Generalize Exercise 4–25 for *n* different toppings. (*Hint:* For example, there are two ways to select pepperoni: either take it or not take it. For two toppings, a person can select none, both, or either one. Continue this reasoning for three toppings, etc.)

*4–27. How many different ways can a person select one or more coins if he has two nickels, one dime, and one half-dollar?

*4–28. A photographer has five photographs that she can mount on a page in her portfolio. How many different ways can she mount her photographs?

*4–29. In a barnyard there is an assortment of chickens and cows. Counting heads, one gets 15; counting legs, one gets 46. How many of each are there?

*4–30. How many committees of two or more people can be formed from four people? (*Hint:* Make a list using the letters A, B, C, and D to represent the people.)

4–3 Permutations and Combinations

Two other rules that can be used to determine the total number of possibilities of a sequence of events are the permutation rule and the combination rule.

Factorial Notation

These rules use *factorial notation.* The factorial notation uses the exclamation point.

$$5! \text{ means } 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

$$9! = 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

Historical Note

In 1808 Christian Kramp first used the factorial notation.

In order to use the formulas in the permutation and combination rules, a special definition of 0! is needed. $0! = 1$.

Factorial Formulas

For any counting n

$$n! = n(n-1)(n-2) \ldots 1$$

$$0! = 1$$

Permutations

A **permutation** is an arrangement of n objects in a specific order.

The next two examples illustrate permutations.

Example 4–6

Suppose a business owner has a choice of five locations in which to establish her business. She decides to rank each location according to certain criteria, such as price of the store and parking facilities. How many different ways can she rank the five locations?

Solution

There are

$$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$$

different possible rankings. The reason is that she has five choices for the first location, four choices for the second location, three choices for the third location, etc.

In the previous example, all objects were used up. But what happens when all objects are not used up? The answer to this question is given in Example 4–7.

Example 4–7

Suppose the business owner in Example 4–6 wishes to rank only the top three of the five locations. How many different ways can she rank them?

Solution

Using the multiplication rule, she can select any one of the five for first choice, then any one of the remaining four locations for her second choice, and finally, any one of the remaining three locations for her third choice, as shown.

First choice		Second choice		Third choice	
5	·	4	·	3	= 60

The solutions in Examples 4–6 and 4–7 are permutations.

Objective 3. Find the number of ways r objects can be selected from n objects using the permutation rule.

Permutation Rule

The arrangement of n objects in a specific order using r objects at a time is called a *permutation of n objects taking r objects at a time.* It is written as ${}_nP_r$ and the formula is

$$ {}_nP_r = \frac{n!}{(n-r)!} $$

The notation ${}_nP_r$ is used for permutations.

$$ {}_6P_4 \text{ means } \frac{6!}{(6-4)!} \text{ or } \frac{6!}{2!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot \cancel{2} \cdot \cancel{1}}{\cancel{2} \cdot \cancel{1}} = 360 $$

Although Examples 4–6 and 4–7 were solved by the multiplication rule, they can now be solved by the permutation rule.

In Example 4–6, five locations were taken and then arranged in order; hence,

$$ {}_5P_5 = \frac{5!}{(5-5)!} = \frac{5!}{0!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1} = 120 $$

(Recall $0! = 1$)

In Example 4–7, three locations were selected from five locations, so $n = 5$ and $r = 3$; hence

$$ {}_5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = \frac{5 \cdot 4 \cdot 3 \cdot \cancel{2} \cdot \cancel{1}}{\cancel{2} \cdot \cancel{1}} = 60 $$

The next two examples illustrate the permutation rule.

Example 4–8

A television news director wishes to use three news stories on an evening show. One story will be the "lead story," one will be the second story, and the last will be a "closing story." If the director has a total of eight stories to choose from, how many possible ways can the program be set up?

Solution

Since order is important, the solution is

$$ {}_8P_3 = \frac{8!}{(8-3)!} = \frac{8!}{5!} = 336 $$

Hence, there would be 336 ways to set up the program.

Example 4–9

How many different ways can a chairperson and an assistant chairperson be selected for a research project if there are seven scientists available?

Solution

$$_7P_2 = \frac{7!}{(7-2)!} = \frac{7!}{5!} = 42$$

Combinations

Objective 4. Find the number of ways *r* objects can be selected from *n* objects without regard to order using the combination rule.

Suppose a dress designer wishes to select two colors of material to design a new dress, and she has on hand four colors. How many different possibilities can there be in this situation?

This type of problem differs from previous ones in that the order of selection is not important. That is, if the designer selects yellow and red, this selection is the same as the selection red and yellow. This type of selection is called a *combination.* The difference between a permutation and a combination is that in a combination, the order or arrangement of the objects is not important; by contrast, order *is* important in a permutation. The next example illustrates this difference.

A selection of distinct objects without regard to order is called a **combination.**

Example 4–10

Given the letters A, B, C, and D, list the permutations and combinations for selecting two letters.

Solution

The listings follow:

Permutations				Combinations	
AB	BA	CA	DA	AB	BC
AC	BC	CB	DB	AC	BD
AD	BD	CD	DC	AD	CD

Interesting Facts

The total number of hours spent mowing lawns in the United States each year: 2,220,000,000. (*The Harper's Index Book*, p. 6)

Note that in permutations, AB is different from BA. But in combinations, AB is the same as BA, so only AB is listed. (Alternatively BA could be listed instead of AB.)

The elements of a combination are usually listed alphabetically.

Combinations are used when the order or arrangement is not important, as in the selecting process. Suppose a committee of 5 students is to be selected from 25 students. The five selected students represent a combination, since it does not matter who is selected first, second, etc.

Combination Rule

The number of combinations of r objects selected from n objects is denoted by $_nC_r$ and is given by the formula

$$_nC_r = \frac{n!}{(n-r)!r!}$$

Example 4–11

How many combinations of four objects are there taken two at a time?

Solution

Since this is a combination problem, the answer is

$$_4C_2 = \frac{4!}{(4-2)!2!} = \frac{4!}{2!2!} = \frac{\overset{2}{\cancel{4}} \cdot 3 \cdot \cancel{2!}}{\cancel{2} \cdot 1 \cdot \cancel{2!}} = 6$$

This is the same result shown in Example 4–10.

Notice that the formula for $_nC_r$ is

$$\frac{n!}{(n-r)!r!}$$

which is the formula for permutations,

$$\frac{n!}{(n-r)!}$$

with an $r!$ in the denominator. This $r!$ divides out the duplicates from the number of permutations, as shown in Example 4–10. For each two letters, there are two permutations but only one combination. Hence, dividing the number of permutations by $r!$ eliminates the duplicates. This result can be verified for other values of n and r. *Note:* $_nC_n = 1$.

Example 4–12

A bicycle shop owner has 12 mountain bicycles in the showroom. The owner wishes to select 5 of them to display at a bicycle show. How many different ways can a group of 5 be selected?

Solution

$$_{12}C_5 = \frac{12!}{(12-5)!5!} = \frac{12!}{7!5!} = \frac{12 \cdot 11 \cdot \overset{2}{\cancel{10}} \cdot \overset{3}{\cancel{9}} \cdot \overset{2}{\cancel{8}} \cdot \cancel{7!}}{\cancel{7!} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1} = 792$$

Example 4–13

In a club there are 7 women and 5 men. A committee of 3 women and 2 men is to be chosen. How many different possibilities are there?

Solution

Here, one must select 3 women from 7 women, which can be done in ${}_7C_3$, or 35, ways. Next, 2 men must be selected from 5 men, which can be done in ${}_5C_2$, or 10, ways. Finally, by the multiplication rule, the total number of different ways is $35 \cdot 10 = 350$, since one is choosing both men and women. Using the formula

$${}_7C_3 \cdot {}_5C_2 = \frac{7!}{(7-3)!3!} \cdot \frac{5!}{(5-2)!2!} = 350$$

Table 4–1 summarizes the counting rules.

Table 4–1 Summary of Counting Rules

Rule	Definition	Formula
Multiplication rule	The number of ways a sequence of n events can occur if the first event can occur in k_1 ways, the second event can occur in k_2 ways, etc.	$k_1 \cdot k_2 \cdot k_3 \cdot \cdots \cdot k_n$
Permutation rule	The number of permutations of n objects taking r objects at a time (order is important)	${}_nP_r = \frac{n!}{(n-r)!}$
Combination rule	The number of combinations of r objects taken from n objects (order is not important)	${}_nC_r = \frac{n!}{(n-r)!r!}$

Exercises

4–31. Evaluate each:
a. 8!
b. 10!
c. 4!
d. 1!
e. 0!

4–32. (ans) Evaluate each expression.
a. ${}_8P_2$
b. ${}_7P_5$
c. ${}_{12}P_4$
d. ${}_5P_3$
e. ${}_6P_0$
f. ${}_6P_6$
g. ${}_8P_0$
h. ${}_8P_8$
i. ${}_{11}P_3$
j. ${}_6P_2$

4–33. How many different four-letter permutations can be formed from the letters in the word *decagon?*

4–34. In a board of directors composed of eight people, how many ways can a chief executive officer, a director, and a treasurer be selected?

4–35. How many different ID cards can be made if there are six digits on a card and no digit can be used more than once?

4–36. How many ways can seven different types of soaps be displayed on a shelf in a grocery store?

4–37. How many different four-color code stripes can be made on a sports car if each code consists of the colors green, red, blue, and white? All colors are used only once.

4–38. An inspector must select three tests to perform in a certain order on a manufactured part. He has a choice of seven tests. How many ways can he perform three different tests?

4–39. The Anderson Research Co. decides to test-market a product in six areas. How many different ways can three areas be selected in a certain order for the first test?

4–40. How many different ways can a city building inspector visit six buildings in the city if she visits all of them in one day?

4–41. How many different ways can five radio commercials be run during an hour of time?

4–42. How many different ways can four tickets be selected from 50 tickets if each ticket wins a different prize?

4–43. How many different ways can a researcher select five rats from 20 rats and assign each to a different test?

4–44. How many different signals can be made by using at least three distinct flags if there are five different flags from which to select?

4–45. An investigative agency has seven cases and five agents. How many different ways can the cases be assigned if only one case is assigned to each agent?

4–46. (ans) Evaluate each expression.

a. ${}_5C_2$	*d.* ${}_6C_2$	*g.* ${}_3C_3$	*j.* ${}_4C_3$
b. ${}_8C_3$	*e.* ${}_6C_4$	*h.* ${}_9C_7$	
c. ${}_7C_4$	*f.* ${}_3C_0$	*i.* ${}_{12}C_2$	

4–47. How many ways can 3 cards be selected from a standard deck of 52 cards disregarding the order of selection?

4–48. How many ways are there to select 3 bracelets from a box of 10 bracelets disregarding the order of selection?

4–49. How many ways can 4 baseball players and 3 basketball players be selected from 12 baseball players and 9 basketball players?

4–50. How many ways can a committee of 4 people be selected from a group of 10 people?

4–51. If a person can select 3 presents from 10 presents under a Christmas tree, how many different combinations are there?

4–52. How many different tests can be made from a test bank of 20 questions if the test consists of 5 questions?

4–53. The general manager of a fast-food restaurant chain must select 6 restaurants from 11 for a promotional program. How many different possible ways can this selection be done?

4–54. How many ways can 3 cars and 4 trucks be selected from 8 cars and 11 trucks to be tested for a safety inspection?

4–55. In a train yard there are 4 tank cars, 12 boxcars, and 7 flatcars. How many ways can a train be made up consisting of 2 tank cars, 5 boxcars, and 3 flatcars? (In this case order is not im portant.)

4–56. There are seven women and five men in a department. How many ways can a committee of four people be selected? How many ways can this committee be selected if there must be two men and two women on the committee? How many ways can this committee be selected if there must be at least two women on the committee?

4–57. Wake Up cereal comes in two types, crispy and crunchy. If a researcher has 10 boxes of each, how many ways can she select 3 boxes of each for a quality control test?

4–58. How many ways can a dinner patron select three appetizers and two vegetables if there are six appetizers and five vegetables on the menu?

4–59. How many ways can a jury of 6 men and 6 women be selected from 12 men and 10 women?

4–60. How many ways can a foursome of 2 men and 2 women be selected from 10 men and 12 women in a golf club?

4–61. The state narcotics bureau must form a 5 member investigative team. If it has 25 agents from which to choose, how many different possible teams can be formed?

4–62. How many different ways can an instructor select 2 textbooks from a possible 17?

4–63. The Environmental Protection Agency must investigate nine mills for complaints of air pollution. How many different ways can a representative select five of these to investigate this week?

4–64. How many ways can a person select 7 television commercials from 11 television commercials?

4–65. How many ways can a person select 8 videotapes from 10 tapes?

4–66. A buyer decides to stock 8 different posters. How many ways can she select these 8 if there are 20 from which to choose?

4–67. An advertising manager decides to have an ad campaign in which 8 special calculators will be hidden at various locations in a shopping mall. If he has 17 locations from which to pick, how many different possible combinations can he choose?

***4–68.** How many different ways can five people—A, B, C, D, and E—sit in a row at a movie theater if (a) A and B must sit together; (b) C must sit to the right of, but not necessarily next to, B; (c) D and E will not sit next to each other?

***4–69.** Using combinations, calculate the number of each poker hand in a deck of cards. (A poker hand consists of five cards dealt in any order.)

a. Royal flush
b. Straight flush
c. Four of a kind
d. Full house

Technology Step by Step

TI-83 Step by Step

Permutations, Combinations, and Factorials

A. To find the value of a permutation: Example ${}_5P_3$
1. Enter **5**.
2. Press **MATH** and mover the cursor to PRB.
3. Press **2**, then **3**, then **ENTER**.

The calculator will display the answer, 60.

B. To find the value of a combination: Example ${}_8C_5$
1. Enter **8**.
2. Press **MATH** move the cursor to PRB, then press 3.
3. Then press **5** and **ENTER**.

The calculator will display the answer, 56.

C. To find a factorial of a number: Example 5!
1. Enter **5**.
2. Press **MATH** and move the cursor to PRB.
3. Press 4, then **ENTER**.

The calculator will display the answer, 120.

4–4 Summary

This chapter illustrates how one can count or list all possible outcomes of a sequence of events. A tree diagram can be used when a list of all possible outcomes is necessary. When only the total number of outcomes is needed, the multiplication rule, the permutation rule, and the combination rule can be used.

Using these rules, statisticians can find the solutions to a variety of problems in which they must know the number of possibilities that can occur. These rules will be used in the next chapter to determine the probabilities of events.

Important Terms

combination 158 permutation 156 tree diagram 151

Important Formulas

Multiplication rule: In a sequence of n events in which the first one has k_1 possibilities, the second event has k_2 possibilities, the third has k_3 possibilities, etc., the total number possibilities of the sequence will be

$$k_1 \cdot k_2 \cdot k_3 \cdot \cdots \cdot k_n$$

Permutation rule: The number of permutations of n objects taking r objects at a time when order is important is

$${}_nP_r = \frac{n!}{(n-r)!}$$

Combination rule: The number of combinations of r objects selected from n objects when order is not important

$${}_nC_r = \frac{n!}{(n-r)!r!}$$

Review Exercises

4–70. An automobile license plate consists of three letters followed by four digits. How many different plates can be made if repetitions are allowed? If repetitions are not allowed? If repetitions are allowed in the letters but not in the digits?

4–71. How many different arrangements of the letters in the word *bread* can be made?

4–72. How many different three-digit combinations can be made by using the numbers 1, 3, 5, 7, and 9 without repetitions if the "right" combination can be used to open a safe? (*Hint:* Does a combination lock really use combinations?)

4–73. How many two-card pairs are there in a poker deck?

4–74. A quiz consists of six multiple-choice questions. Each question has three possible answer choices. How many different answer keys can be made?

4–75. How many ways can 5 different television programs be selected from 12 programs?

4–76. How many different ways can a buyer select four television models from a possible choice of six models?

4–77. Draw a tree diagram to show all possible outcomes when a coin is flipped four times.

4–78. How many ways can three outfielders and four infielders be chosen from five outfielders and seven infielders?

4–79. How many different ways can eight computer operators be seated in a row?

4–80. How many ways can a student select 2 electives from a possible choice of 10 electives?

4–81. There are six Republican, five Democrat, and four Independent candidates. How many different ways can a committee of three Republicans, two Democrats, and one Independent be selected?

4–82. Using Exercise 4–81, how many ways can a committee of four people be selected if they are all from the same party?

4–83. Employees can be classified according to gender (male, female), income (low, medium, high), and rank (assistant, instructor, dean). Draw a tree diagram and show all possible outcomes.

4–84. A disc jockey can select 4 records from 10 to play in one segment. How many ways can this selection be done? (*Note:* Order is important.)

4–85. A judge is to rank six brands of cookies according to their flavor. How many different ways can this ranking be done?

4–86. A vending machine servicer must restock and collect money from 20 machines, each one at a different location. How many different ways can he select 5 machines to service in one day?

4–87. How many different ways can four floral centerpieces be arranged in a display case? (Assume four spaces are available.)

4–88. How many different ways can 3 fraternity members be selected from 10 members if one must be president, one must be vice president, and one must be secretary/treasurer?

4–89. How many different ways can two balls be drawn from a bag containing five balls? Each ball is a different color, and the first ball is replaced before the second one is selected. How many different ways are there if the first ball is not replaced before the second one is selected?

4–90. A restaurant offers three choices of meat, two choices of potatoes, four choices of vegetables, and five choices of dessert. How many different possible meals can be made if a customer must select one item from each category?

4–91. If someone wears a blouse or a sweater and a pair of slacks or a skirt, how many different outfits can she wear?

4–92. How many different computer passwords are possible if each consists of four symbols and if the first one must be a letter and the other three must be digits?

4–93. If a student has a choice of five computers, three printers, and two monitors, how many ways can she select a computer system?

4–94. A combination lock consists of the numbers 0 to 39. If no number can be used twice, how many different combinations are possible using three numbers? Remember, a combination lock is really a permutation lock.

4–95. There are 12 students who wish to enroll in a particular course. There are only four seats left in the classroom. How many different ways can 4 students be selected to attend the class?

4–96. A candy store allows customers to select three different candies to be packaged and mailed. If there are 13 varieties available, how many possible selections can be made?

4–97. If a student can select 5 novels from a reading list of 20 for a course in literature, how many different possible ways can this selection be done?

4–98. If a student can select one of three language courses, one of five mathematics courses, and one of four history courses, how many different schedules can be made?

Statistics Today

Why Are We Running Out of 800 Numbers? Revisited

Since each 800 number is followed by a seven-digit number, multiplication rule 1 can be used to determine the total number of 800 phone numbers that are available.

Since there are 10 digits (0 through 9) that can be used for each digit of the seven-digit number, the answer is 10^7, or 10,000,000, starting with 800-0000000 and ending with 800-9999999. (Since the telephone industry forbids usage of certain numbers, the article that opens this chapter refers to a smaller number of "possible" 800 numbers.)

Chapter Quiz

Determine whether each statement is true or false. If the statement is false, explain why.

1. If there are 5 contestants in a race, the number of different ways the first- and second-place winners can be selected is 25.
2. If a true–false exam contains 10 questions, there are 20 different ways to answer all the questions.
3. Some permutation problems can be solved by the multiplication rule.
4. The arrangement ABC is the same as BAC for combinations.
5. When objects are arranged in a specific order, the arrangement is called a combination.

Select the best answer.

6. What is ${}_nP_0$?
   a. 0
   b. 1
   c. n
   d. It cannot be determined.
7. What is the number of permutations of six different objects taken all together?
   a. 0 *c.* 36
   b. 1 *d.* 720
8. How many permutations of the letters in the word *tide* are there?
   a. 1 *c.* 12
   b. 6 *d.* 24
9. What is 0!?
   a. 0 *c.* undefined
   b. 1 *d.* 10
10. What is ${}_nC_n$?
    a. 0 *c.* n
    b. 1 *d.* It cannot be determined.

Complete the following statements with the best answer.

11. A device that is helpful in listing the outcomes of a sequence of events is called a ______.
12. When a coin is tossed and a die is rolled, there are ______ outcomes.
13. If in a sequence of k events each event can occur the same number of ways (n), then the total number of outcomes is ______.
14. The number of permutations of n objects taken all together is ______.
15. Telephone numbers are examples of ______.
16. One company's ID cards consist of five letters followed by two digits. How many cards can be made if repetitions are allowed? If repetitions are not allowed?
17. How many different arrangements of the letters in the word *number* can be made?
18. A physics test consists of 25 true–false questions. How many different possible answer keys can be made?
19. How many different ways can four radios be selected from a total of seven radios?
20. The National Bridge Association can select one of four cities for its playoff tournament next year. The cities are Pasadena, Wilmington, Chicago, and Charleston. The following year, it can hold the tournament in Hyattsville or Green Springs. How many different possibilities are there for the next two years? Draw a tree diagram and show all possibilities.
21. How many ways can five sopranos and four altos be selected from seven sopranos and nine altos?
22. How many different ways can eight speakers be seated on a stage?
23. Employees can be classified according to gender (male, female), income (low, medium, high), and rank (staff nurse, charge nurse, head nurse). Draw a tree diagram and show all possible outcomes.
24. A soda machine servicer must restock and collect money from 15 machines, each one at a different location. How many ways can she select four machines to service in one day?