Chapter 4

Counting Techniques

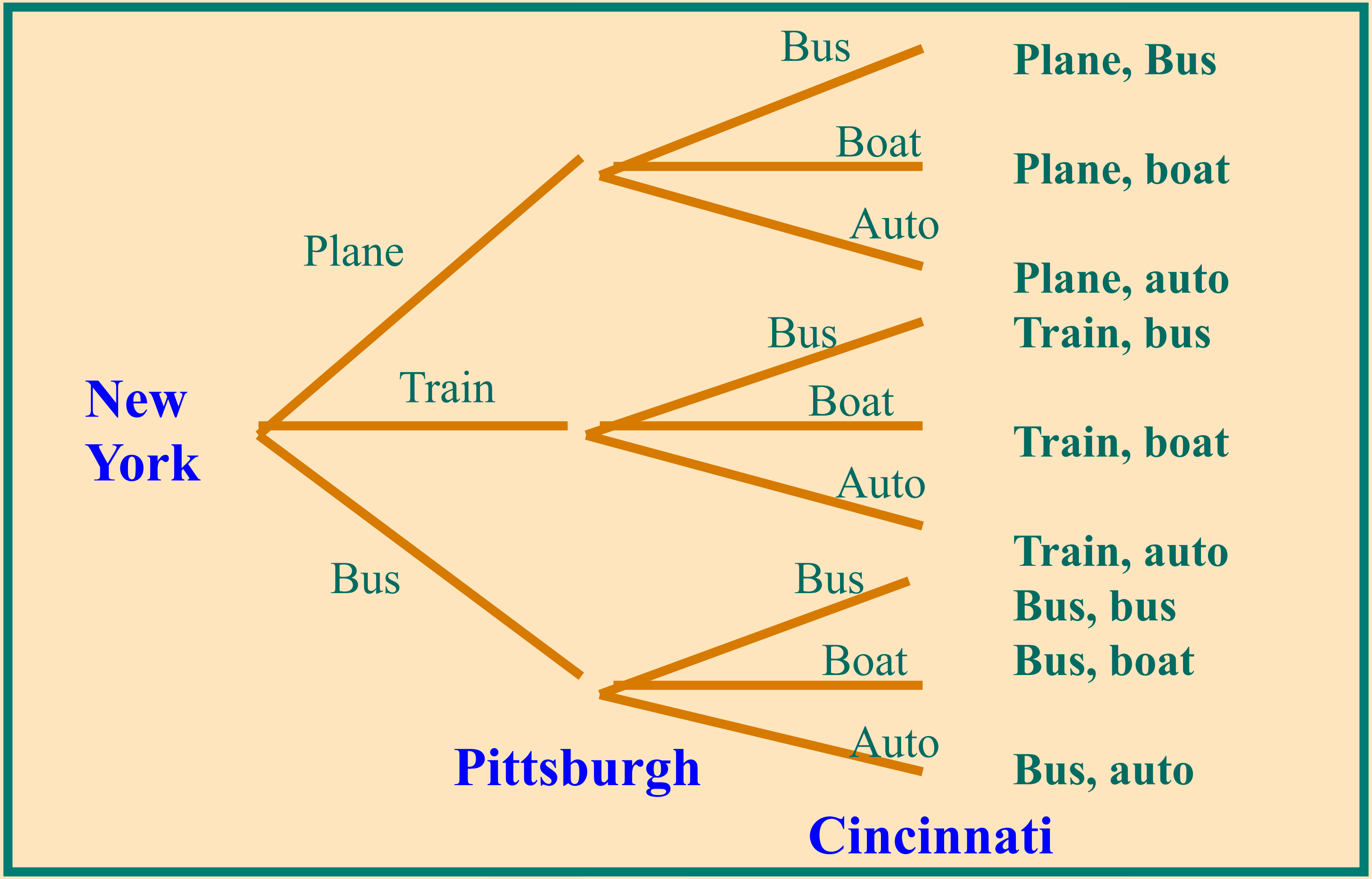
4-2 Tree Diagrams

- A **tree diagram** is a device used to list all possibilities of a sequence of events in a systematic way.

4-2 Tree Diagrams - Example

- Suppose a sales person can travel from New York to Pittsburgh by plane, train, or bus, and from Pittsburgh to Cincinnati by bus, boat, or automobile. Display the information using a tree diagram.

4-2 Tree Diagrams - Example



4-2 The Multiplication Rule for Counting

- **Multiplication Rule :** In a sequence of n events in which the first one has k_1 possibilities and the second event has k_2 and the third has k_3, and so forth, the total possibilities of the sequence will be $k_1 \times k_2 \times k_3 \times \cdots \times k_n$.

4-2 The Multiplication Rule for Counting - Example

- A nurse has three patients to visit. How many different ways can she make her rounds if she visits each patient only once?

4-2 The Multiplication Rule for Counting - Example

- She can choose from three patients for the first visit and choose from two patients for the second visit, since there are two left. On the third visit, she will see the one patient who is left. Hence, the total number of different possible outcomes is $3 \times 2 \times 1 = 6$.

4-2 The Multiplication Rule for Counting - Example

- Employees of a large corporation are to be issued special coded identification cards. The card consists of 4 letters of the alphabet. Each letter can be used up to 4 times in the code. How many different ID cards can be issued?

4-2 The Multiplication Rules for Counting - Example

- Since 4 letters are to be used, there are 4 spaces to fill (_ _ _ _). Since there are 26 different letters to select from and each letter can be used up to 4 times, then the total number of identification cards that can be made is $26 \times 26 \times 26 \times 26 = 456{,}976$.

4-2 The Multiplication Rule for Counting - Example

- The digits 0, 1, 2, 3, and 4 are to be used in a 4-digit ID card. How many different cards are possible if repetitions are permitted?
- *Solution:* Since there are four spaces to fill and five choices for each space, the solution is $5 \times 5 \times 5 \times 5 = 5^4 = 625$.

4-2 The Multiplication Rule for Counting - Example

- What if the repetitions were not permitted in the previous example?
- *Solution:* The first digit can be chosen in five ways. But the second digit can be chosen in only four ways, since there are only four digits left; etc. Thus the solution is $5 \times 4 \times 3 \times 2 = 120$.

4-3 Permutations

- Consider the possible arrangements of the letters *a*, *b*, and *c*.
- The possible arrangements are: ***abc, acb, bac, bca, cab, cba.***
- If the ***order of the arrangement is important*** then we say that each arrangement is a permutation of the three letters. Thus there are six permutations of the three letters.

4-3 Permutations

- An arrangement of *n* distinct objects in a specific order is called a **permutation** of the objects.
- ***Note:*** To determine the number of possibilities mathematically, one can use the multiplication rule to get: $3 \times 2 \times 1 = 6$ permutations.

4-3 Permutations

- **Permutation Rule :** The arrangement of n objects in a specific order using r objects at a time is called a permutation of n objects taken r objects at a time. It is written as ${}_nP_r$ and the formula is given by ${}_nP_r = n! / (n - r)!$.

4-3 Permutations - Example

- How many different ways can a chairperson and an assistant chairperson be selected for a research project if there are seven scientists available?
- ***Solution:*** Number of ways $= {}_7P_2 = 7!/(7-2)! = 7!/5! = 42$.

4-3 Permutations - Example

- How many different ways can four books be arranged on a shelf if they can be selected from nine books?
- ***Solution:*** Number of ways $= {}_9P_4 = 9! / (9-4)! = 9!/5! = 3024$.

4-3 Combinations

- Consider the possible arrangements of the letters *a*, *b*, and *c*.
- The possible arrangements are: *abc, acb, bac, bca, cab, cba.*
- If the *order of the arrangement is not important* then we say that each arrangement is the same. We say there is one combination of the three letters.

4-3 Combinations

- **Combination Rule :** The number of combinations of of r objects from n objects is denoted by ${}_nC_r$ and the formula is given by ${}_nC_r = n! / [(n - r)!r!]$.

4-3 Combinations - Example

- How many combinations of four objects are there taken two at a time?
- *Solution:* Number of combinations: $_4C_2 = 4! / [(4 - 2)!\ 2!] = 4!/[2!2!] = 6.$

4-3 Combinations - Example

- In order to survey the opinions of customers at local malls, a researcher decides to select 5 malls from a total of 12 malls in a specific geographic area. How many different ways can the selection be made?
- ***Solution:*** Number of combinations: $_{12}C_5 = 12! / [(12 - 5)!\ 5!] = 12!/[7!5!] = 792.$

4-3 Combinations - Example

- In a club there are 7 women and 5 men. A committee of 3 women and 2 men is to be chosen. How many different possibilities are there?
- *Solution:* Number of possibilities: (number of ways of selecting 3 women from 7) × (number of ways of selecting 2 men from 5) = ${}_7C_3 \times {}_5C_2 = (35)(10) = 350$.

4-3 Combinations - Example

- A committee of 5 people must be selected from 5 men and 8 women. How many ways can the selection be made if there are at least 3 women on the committee?

4-3 Combinations - Example

- **Solution:** The committee can consist of 3 women and 2 men, or 4 women and 1 man, or 5 women. To find the different possibilities, find each separately and then add them: $_8C_3 \times {_5C_2} + {_8C_4} \times {_5C_1} + C_5 \times {_5C_0} = (56)(10) + (70)(5) + (56)(1) = 966.$