

$$(1, 4) \quad (2, 12) \quad y = ab^x$$

$$4 = ab^1$$

$$(2, 12) \quad a = \frac{4}{b}$$

$$\frac{4}{b} = \frac{ab}{b}$$

$$12 = \frac{4}{b} b^2 \rightarrow 12 = \frac{4b^2}{b}$$

$$a = \frac{4}{b}$$

$$\frac{12}{4} = \frac{4b}{4} \quad a = \frac{4}{3}$$

$$\boxed{b = 3}$$

$$y = \frac{4}{3} 3^x$$

Summary

Families of Exponential Functions

Parent function:

$$y = b^x$$

Stretch ($|a| > 1$)

Shrink ($0 < |a| < 1$)

Reflection ($a < 0$) in x -axis

}

$$y = ab^x$$

Translation (horizontal by h ; vertical by k):

$$y = b^{x-h} + k$$

Combined:

$$y = ab^{x-h} + k$$

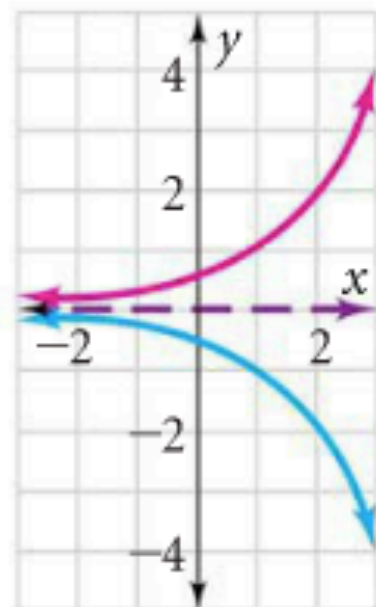
1 EXAMPLE Graphing $y = ab^x$ for $0 < |a| < 1$

Graph $y = \frac{1}{2} \cdot 2^x$ and $y = -\frac{1}{2} \cdot 2^x$. Label the asymptote of each graph.

Step 1 Make a table of values.

x	$y = \frac{1}{2} \cdot 2^x$	$y = -\frac{1}{2} \cdot 2^x$
-2	$\frac{1}{8}$	$-\frac{1}{8}$
-1	$\frac{1}{4}$	$-\frac{1}{4}$
0	$\frac{1}{2}$	$-\frac{1}{2}$
1	1	-1
2	2	-2
3	4	-4

Step 2 Graph the functions.



The y -intercept is a , or $\frac{1}{2}$.

The asymptote is $y = 0$ for both graphs.

The y -intercept is a , or $-\frac{1}{2}$.

$y = \frac{1}{2} \cdot 2^x$ shrinks $y = 2^x$ by a factor of $\frac{1}{2}$.

$y = -\frac{1}{2} \cdot 2^x$ reflects $y = \frac{1}{2} \cdot 2^x$ in the x -axis.

2 EXAMPLE Translating $y = ab^x$

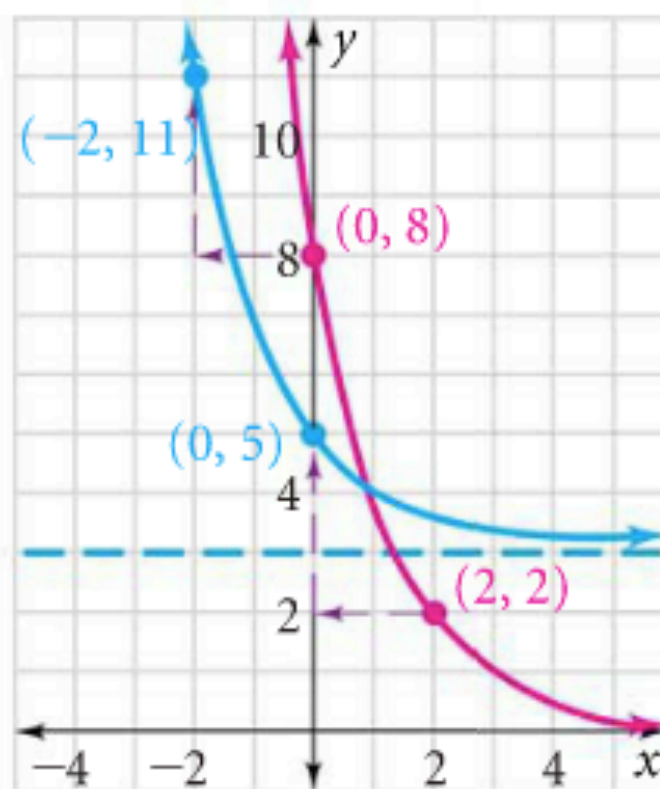
Graph the stretch $y = 8\left(\frac{1}{2}\right)^x$ and then the translation $y = 8\left(\frac{1}{2}\right)^{x+2} + 3$.

Step 1 Graph $y = 8\left(\frac{1}{2}\right)^x$. The horizontal asymptote is $y = 0$.

Step 2 For $y = 8\left(\frac{1}{2}\right)^{x+2} + 3$, $h = -2$ and $k = 3$. So shift the $y = 8\left(\frac{1}{2}\right)^x$ graph 2 units left and 3 units up. The horizontal asymptote is $y = 3$.

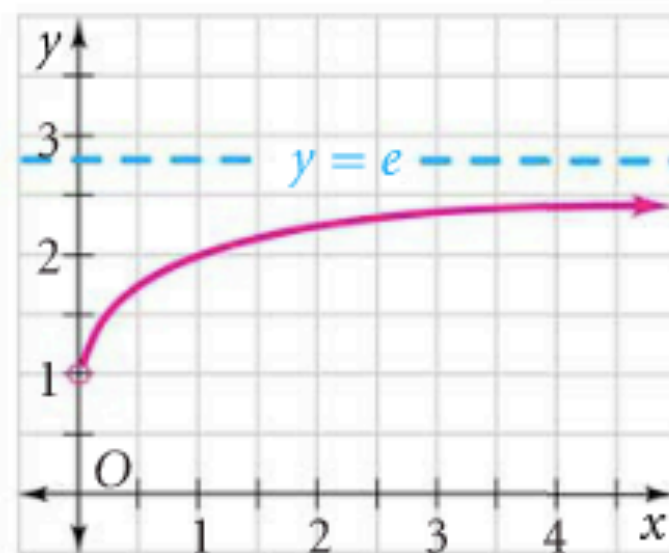
2 Graph the stretch $y = 9(3)^x$ and then each translation.

- a. $y = 9(3)^{x+1}$
- b. $y = 9(3)^x - 4$
- c. $y = 9(3)^{x-3} - 1$



At the right is part of the graph of the function $y = \left(1 + \frac{1}{x}\right)^x$. One of the graph's asymptotes is $y = e$, where e is an irrational number approximately equal to 2.71828.

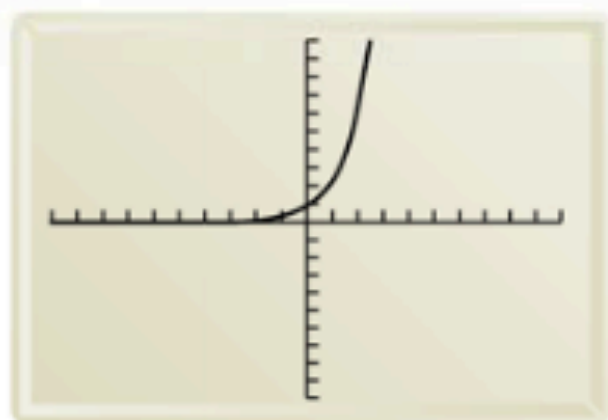
Exponential functions with a base of e are useful for describing continuous growth or decay. Your graphing calculator has a key for e^x .



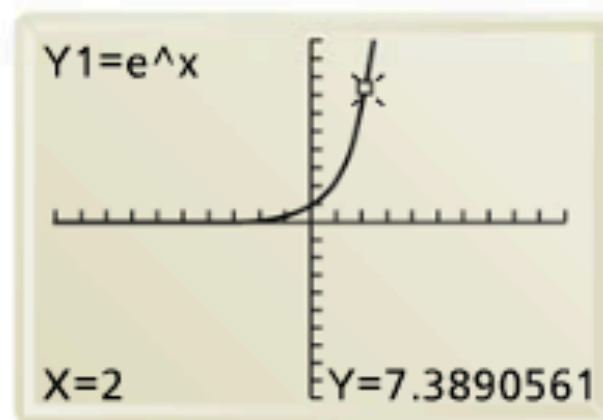
4 EXAMPLE Evaluating e^x

Graph $y = e^x$. Evaluate e^2 to four decimal places.

Step 1 Graph $y = e^x$.



Step 2 Find y when $x = 2$.



Graph each function as a transformation of its parent function.

9. $y = 8^x + 5$

10. $y = 15\left(\frac{4}{3}\right)^x - 8$

11. $y = -(0.3)^{x-2}$

12. $y = -2(5)^{x+3}$

13. $y = 52\left(\frac{2}{13}\right)^{x-1} + 26$

14. $y = 9\left(\frac{1}{3}\right)^{x+7} - 3$

Use the graph of $y = e^x$ to evaluate each expression to four decimal places.

18. e^3

19. e^6

20. e^{-2}

21. e^0

22. $e^{\frac{5}{2}}$

23. e^e

