Absolute Value Functions and Transformations

Objectives:

1. To graph an absolute value function by performing transformations on the parent

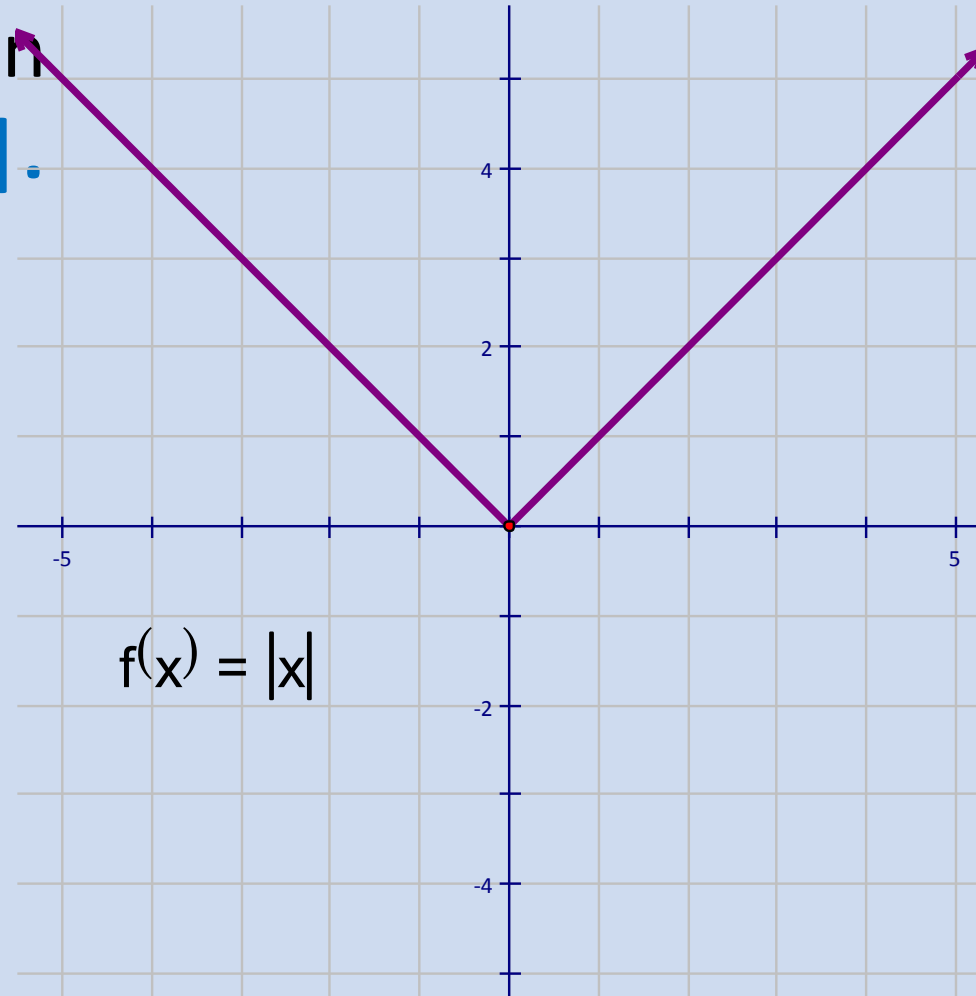# Vocabulary

Try to define each of these terms. Give an example of each word and leave a bit of space for additions and revisions.
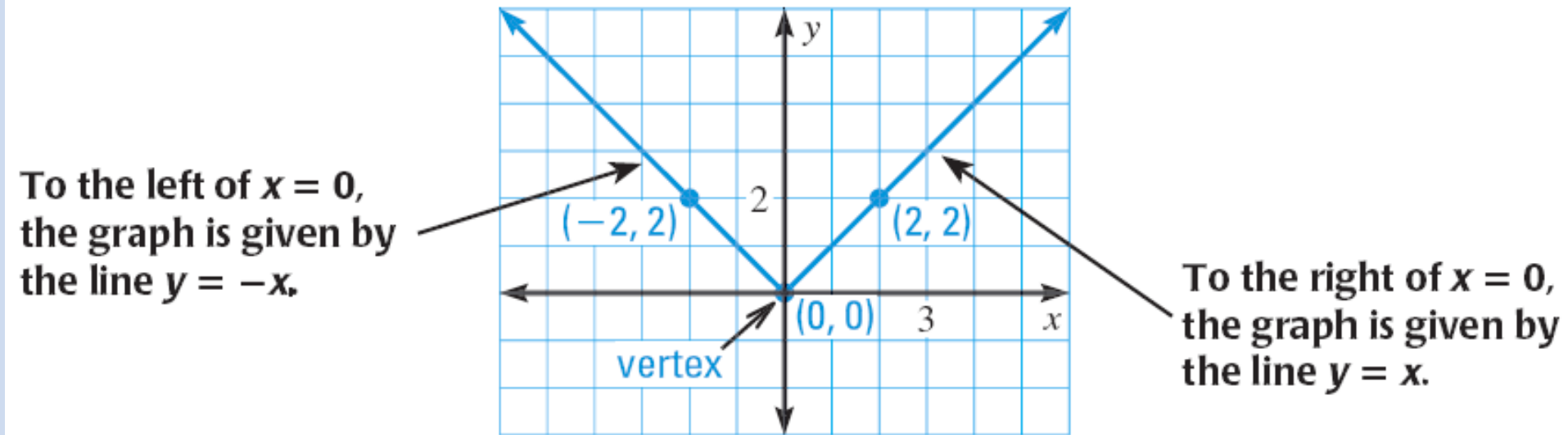
Transformation	Translation
Reflection	

The Absolute Value Function

The absolute value function is defined by $f(x) = |x|$.

This is the absolute value **parent function**.

$f(x) = |x|$

Parent Function

To the left of $x = 0$, the graph is given by the line $y = -x$.

$(-2, 2)$

$(2, 2)$

$(0, 0)$

vertex

To the right of $x = 0$, the graph is given by the line $y = x$.

- V-shape
- It is symmetric about the y-axis
- The **vertex** is the minimum point on the graph

Investigation 1

In this Investigation, we are going to discover how to perform transformations on the absolute value function.
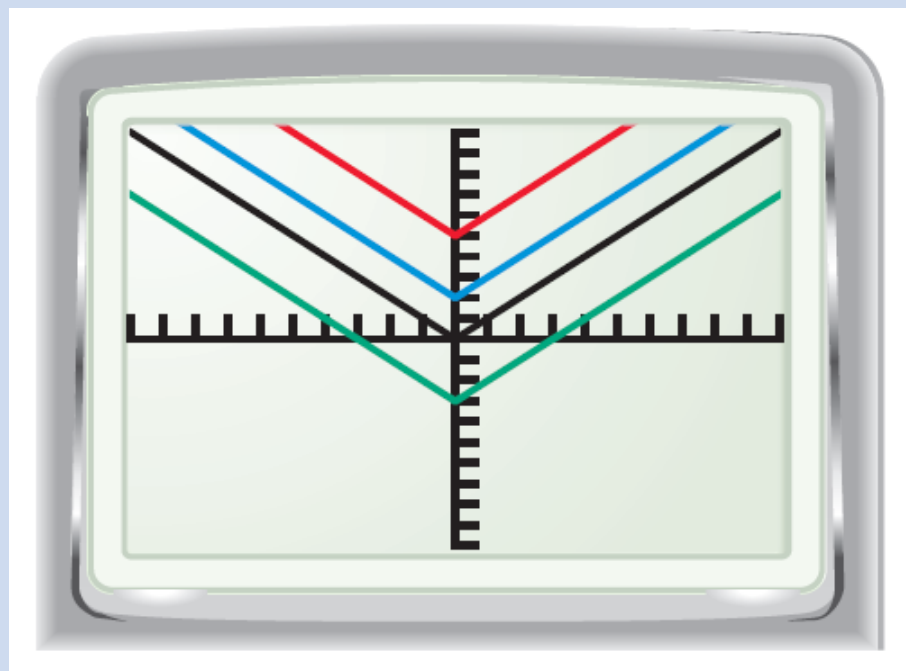
Step 1: Press Y= and enter the following equations.

Y1= abs(X)	Y3= abs(X) + 5
Y2= abs(X)+2	Y4= abs(X) − 3

Investigation 1

Step 2: Graph the equations using the ZOOMSTD setting.

Step 3: Describe how the family of graphs $y = |x| + k$ is related to $y = |x|$.
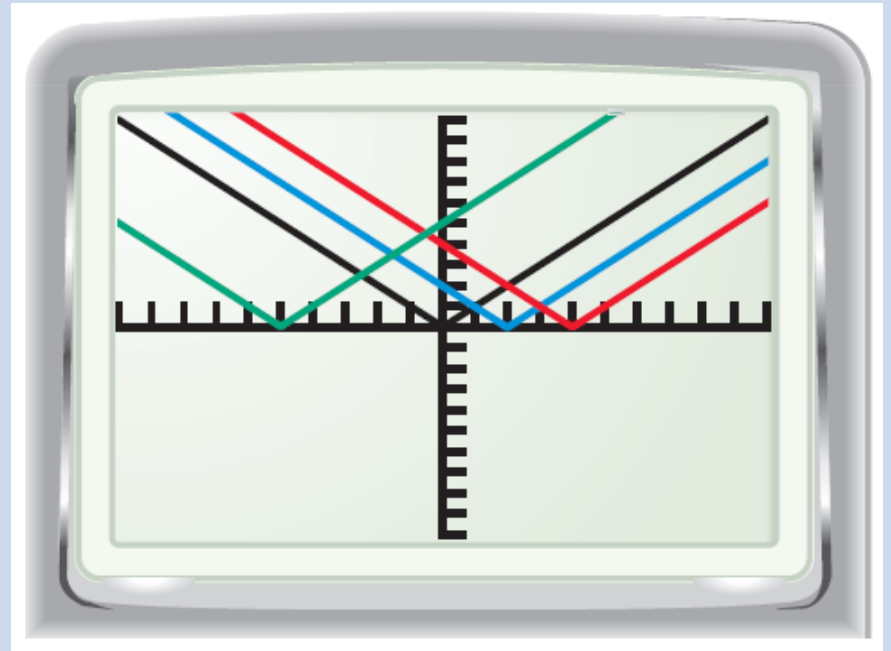
Investigation 1

Step 4: Press Y= and enter the following equations.

Y1= abs(X)	Y3= abs(X – 4)
Y2= abs(X – 2)	Y4= abs(X + 5)

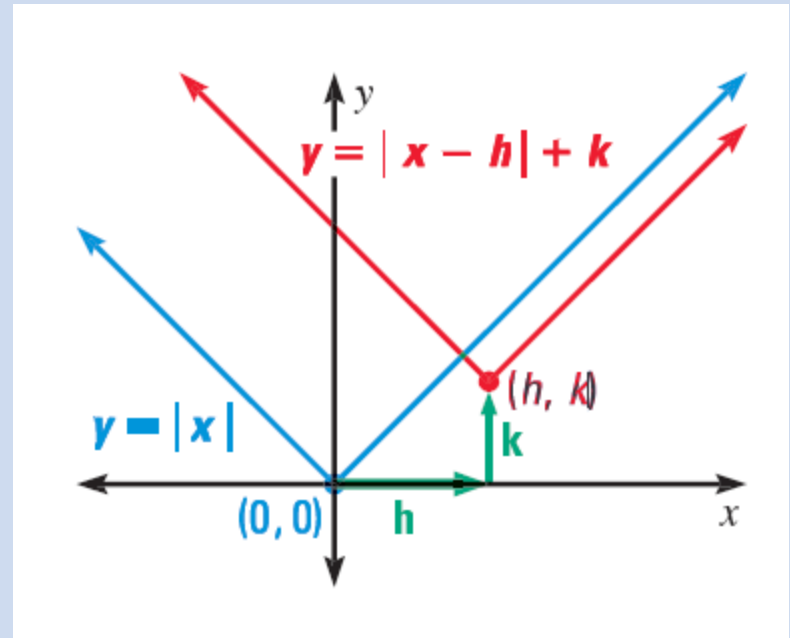**Step 5:** Graph the equations using the ZOOMSTD setting.

Investigation 1

Step 6: Describe how the family of graphs $y = |x - h|$ is related to $y = |x|$.

Translation

A **translation** is a transformation that shifts a graph horizontally or vertically, but doesn't change the overall shape or orientation.
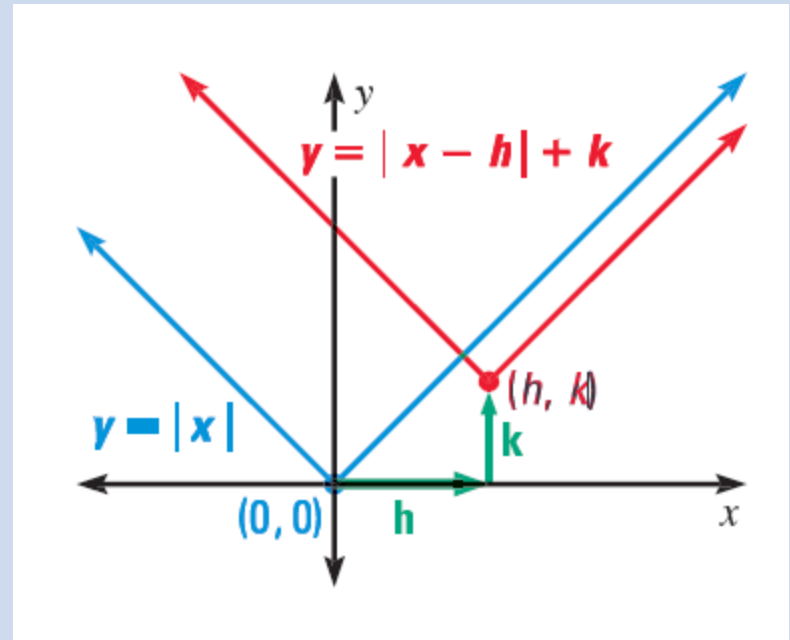
Translation

The graph of

$$y = |x - h| + k$$

is the graph of $y = |x|$ translated h horizontal units and y vertical units.

- **The new vertex is at (h, k)**

Investigation 2

In this Investigation, we will continue to expand our knowledge of transformations.

Step 1: Press Y= and enter the following equations.

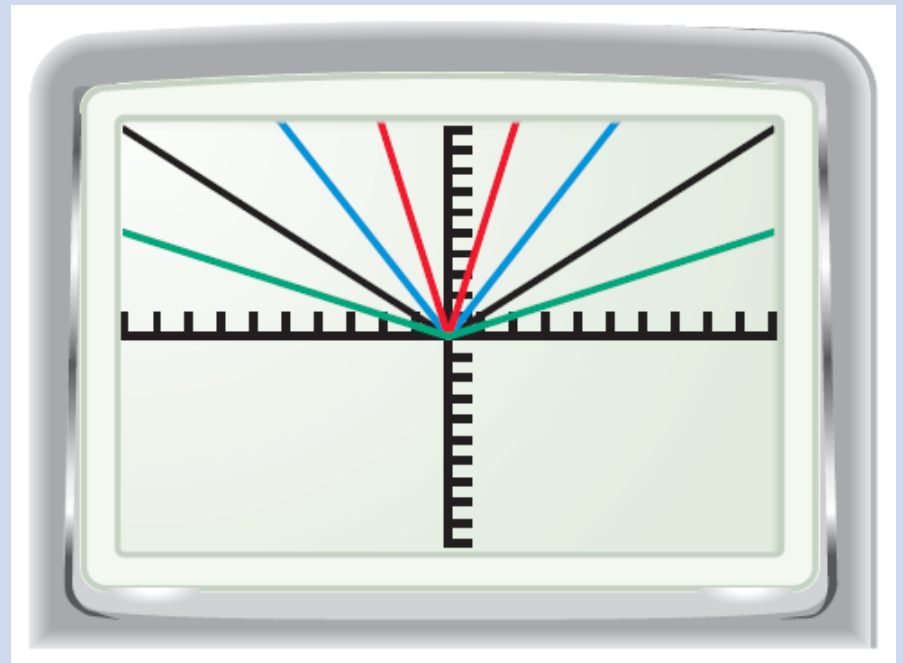|	
Y1= abs(X)	Y3= 2*abs(X)
Y2= 5*abs(X)	Y4= (1/2)*abs(X)

Investigation 2

Step 2: Graph the equations using the ZOOMSTD setting.

Step 3: Describe how the family of graphs $y = a|x|$ is related to $y = |x|$.

Stretching and Shrinking

The graph of $y = a|x|$ is graph of $y = |x|$ vertically stretched or shrunk depending on the $|a|$.

For $|a| > 1$	For $|a| < 1$
• The graph is vertically *stretched*, or elongated.	• The graph is vertically *shrunk*, or compressed.
• The graph of $y = a|x|$ is *narrower* than the graph of $y = |x|$.	• The graph of $y = a|x|$ is *wider* than the graph of $y = |x|$.

Exercise 1

Use your graphing calculator to graph the following:

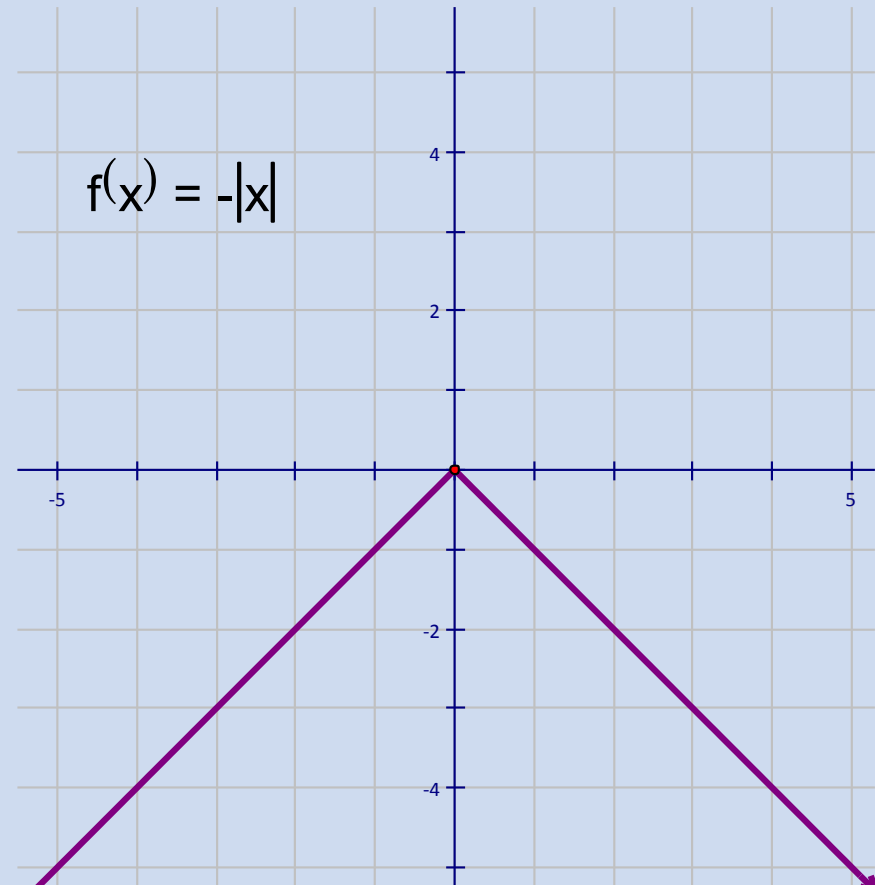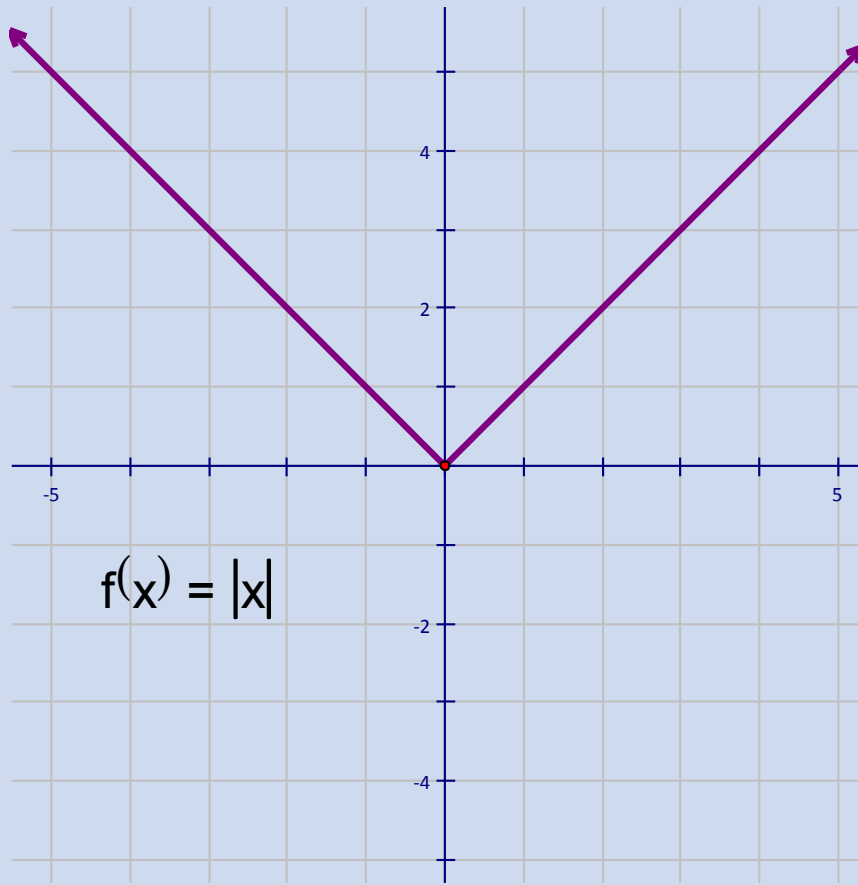1. y = |x|
2. y = -|x|

Describe how the graph of $y = a|x|$ is related to $y = |x|$ when $a < 0$.

Reflection

The graph of $y = a|x|$ is graph of $y = |x|$ reflected across the x-axis when $a < 0$.

$f(x) = |x|$

$f(x) = -|x|$

Multiple Transformations

In general, the graph of an absolute value function of the form $y = a|x - h| + k$ can involve translations, reflections, stretches or shrinks.

To graph an absolute value function, start by identifying the vertex.

Graphing Absolute Value Functions

Graphing $y = a|x - h| + k$ these things is easy:

1. Plot the vertex (h, k).
2. Use the a value as slope to plot one point.
3. Use symmetry to find a corresponding point.
4. Connect the dots in a V-shape.

Exercise 2

Without a graphing calculator, graph the following functions. How do they compare to the parent function?

1. $y = |x - 2| + 5$

2. $y = (1/2)|x|$

3. $y = 2|x + 1| - 3$

4. $f(x) = \text{-}3|x + 1| - 2$

Transformations in General

You can perform transformations on the graph of any function in manner similar to transformations on the absolute value function.

The graph of $y = a \cdot f(x - h) + k$ can be obtained from the graph of any function $y = f(x)$ by performing these steps:

STEP 1 **Stretch or shrink** the graph of $y = f(x)$ vertically by a factor of $|a|$ if $|a| \neq 1$. If $|a| > 1$, stretch the graph. If $|a| < 1$, shrink the graph.

STEP 2 **Reflect** the resulting graph from Step 1 in the x-axis if $a < 0$.

STEP 3 **Translate** the resulting graph from Step 2 horizontally h units and vertically k units.