

## Graphing Logs the Easy Way:

1) Find two points.

Remember that  $\log_a 1 = 0$   
 $\log_a a = 1$

2) Find vert asymptote.

$x =$  horiz shift

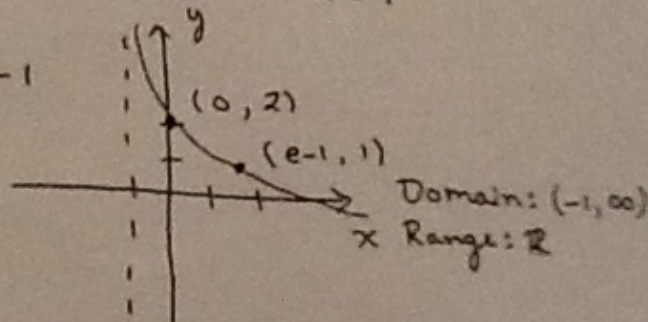
3) Sketch the log shape.

Ex: Sketch a graph of  $f(x) = -\ln(x+1) + 2$

1) Two Points:

$x$	$f(x)$
0	$-\ln(1) + 2 = 2$
$e-1$	$-\ln(e) + 2 = -1 + 2 = 1$

2) Vert Asymptote:  $x = -1$



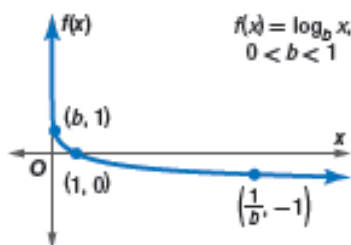
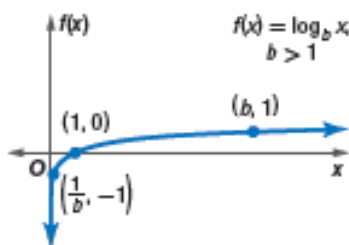
**2 Graphing Logarithmic Functions** The function  $y = \log_b x$ , where  $b \neq 1$ , is called a **logarithmic function**. The graph of  $f(x) = \log_b x$  represents a parent graph of the logarithmic functions.

**KeyConcept** Parent Function of Logarithmic Functions



Parent function:  $f(x) = \log_b x$   
 Domain: all positive real numbers  
 Asymptote:  $f(x)$ -axis

Type of graph: continuous, one-to-one  
 Range: all real numbers  
 Intercept:  $(1, 0)$



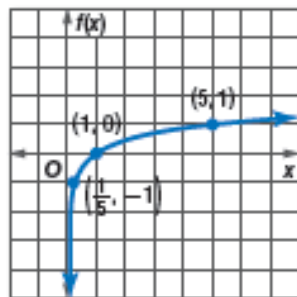
Graph each function.

a.  $f(x) = \log_5 x$

**Step 1** Identify the base.  
 $b = 5$

**Step 2** Determine points on the graph.  
 Because  $5 > 1$ , use the points  $(\frac{1}{5}, -1)$ ,  $(1, 0)$ , and  $(5, 1)$ .

**Step 3** Plot the points and sketch the graph.  
 $(\frac{1}{5}, -1) \rightarrow (\frac{1}{5}, -1)$   
 $(1, 0)$   
 $(b, 1) \rightarrow (5, 1)$

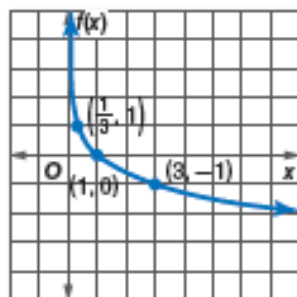


b.  $f(x) = \log_{\frac{1}{3}} x$

**Step 1**  $b = \frac{1}{3}$

**Step 2**  $0 < \frac{1}{3} < 1$ , so use the points  
 $(\frac{1}{3}, 1)$ ,  $(1, 0)$  and  $(3, -1)$ .

**Step 3** Sketch the graph.



## KeyConcept Transformations of Logarithmic Functions

$$f(x) = a \log_b (x - h) + k$$

### $h$ – Horizontal Translation

$h$  units right if  $h$  is positive  
 $|h|$  units left if  $h$  is negative

### $k$ – Vertical Translation

$k$  units up if  $k$  is positive  
 $|k|$  units down if  $k$  is negative

### $a$ – Orientation and Shape

If  $a < 0$ , the graph is reflected across the  $x$ -axis.

If  $|a| > 1$ , the graph is stretched vertically.

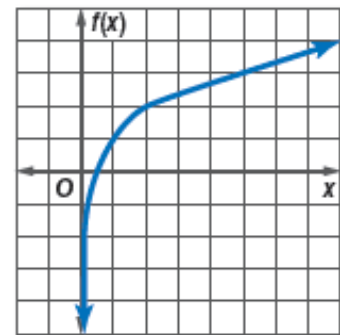
If  $0 < |a| < 1$ , the graph is compressed vertically.

Graph each function.

a.  $f(x) = 3 \log_{10} x + 1$

This represents a transformation of the graph of  $f(x) = \log_{10} x$ .

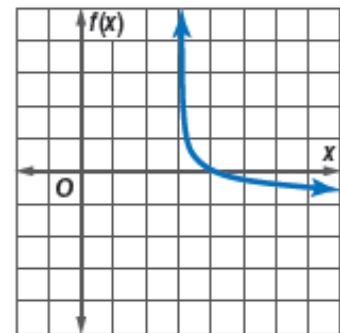
- $|a| = 3$ : The graph stretches vertically
- $h = 0$ : There is no horizontal shift.
- $k = 1$ : The graph is translated 1 unit up.



b.  $f(x) = \frac{1}{2} \log_{\frac{1}{4}} (x - 3)$

This is a transformation of the graph of  $f(x) = \log_{\frac{1}{4}} x$ .

- $|a| = \frac{1}{2}$ : The graph is compressed vertically.
- $h = 3$ : The graph is translated 3 units to the right.
- $k = 0$ : There is no vertical shift.

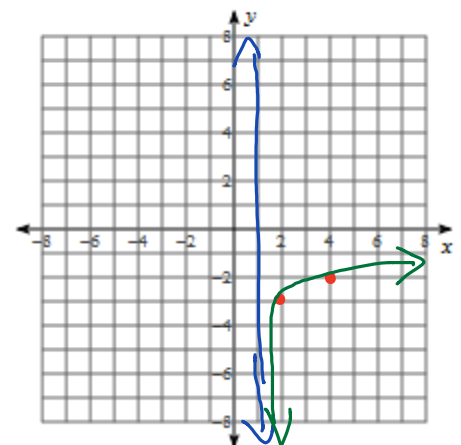


## Example

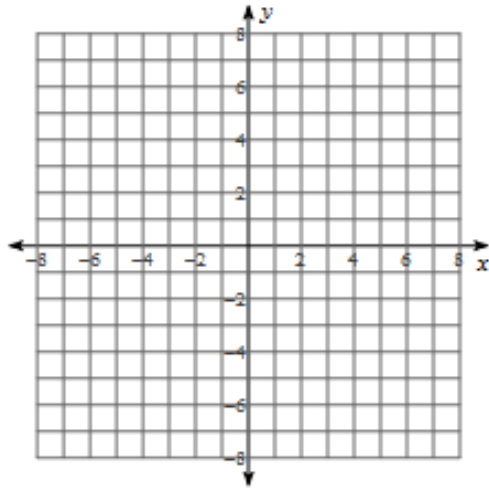
$\log_3 x$	$\log_3 (x-1)$ Shift 1 right	$\log_3 (x-1) - 3$ Shift down 3
(1, 0)	(2, 0)	(2, -3)
(3, 1)	(4, 1)	(4, -2)

Vert asymptote = Vertical shift  
 $x = 1$

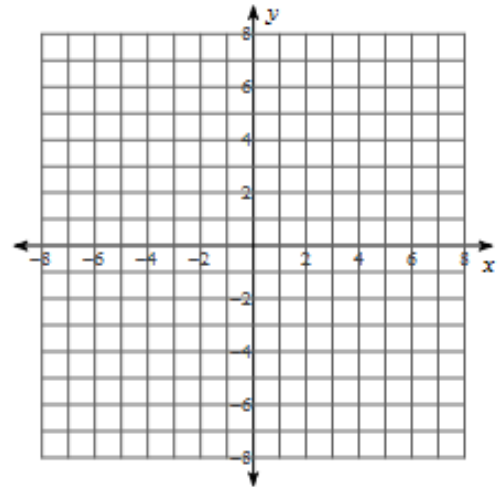
1)  $y = \log_3 (x - 1) - 3$



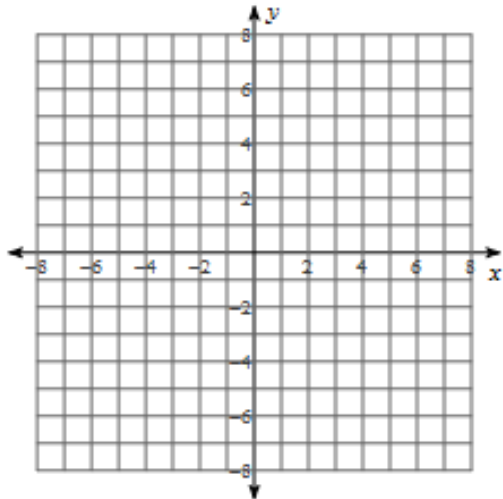
$$1) y = \log_5 (x - 1) + 5$$



$$2) y = \log (x - 1) - 3$$



$$3) y = \log (x - 1) - 4$$



$$4) y = \log_6 (x - 1) - 3$$

