Factoring Polynomials

A “**Difference of Squares**” is a *binomial* (***2 terms only***) and it factors like this:

$$a^2 - b^2 = (a + b)(a - b)$$

Factoring a polynomial means expressing it as a *product* of other polynomials.

Factoring Method #1

Factoring polynomials with a common *monomial* factor (using GCF).

****Always look for a GCF before using any other factoring method.**

Steps:

1. Find the greatest common factor (GCF).
2. Divide the polynomial by the GCF. The quotient is the other factor.
3. Express the polynomial as the product of the quotient and the GCF.

$$\textbf{\textit{Example}}: \ 6\boldsymbol{c}^3\boldsymbol{d} - 12\boldsymbol{c}^2\boldsymbol{d}^2 + 3\boldsymbol{cd}$$

Step 1: $\boldsymbol{GCF} = 3\boldsymbol{cd}$

Step 2: ***Divide by GCF***

$$(6c^3d - 12c^2d^2 + 3cd) \div 3cd =$$

$$2c^2 - 4cd + 1$$

The answer should look like this:

$$Ex: \quad 6c^3d - 12c^2d^2 + 3cd$$

$$= \boxed{3cd(2c^2 - 4cd + 1)}$$

Factor these on your own looking for a GCF.

1. $6x^3 + 3x^2 - 12x$ $= 3x\left(2x^2 + x - 4\right)$

2. $5x^2 - 10x + 35$ $= 5\left(x^2 - 2x + 7\right)$

3. $16x^3y^4z - 8x^2y^2z^3 + 12xy^3z^2$

$$= 4xy^2z\left(4x^2y^2 - 2xz^2 + 3yz\right)$$

Factoring Method #2

Factoring polynomials that are a *difference of squares*.

To factor, express each term as a square of a monomial then apply the rule... $a^2 - b^2 = (a+b)(a-b)$

$$Ex: \quad x^2 - 16 =$$

$$x^2 - 4^2 =$$

$$\boxed{(x+4)(x-4)}$$

Here is another example:

$$\frac{1}{49}x^2 - 81 =$$

$$\left(\frac{1}{7}x\right)^2 - 9^2 = \boxed{\left(\frac{1}{7}x + 9\right)\left(\frac{1}{7}x - 9\right)}$$

Try these on your own:

1. $x^2 - 121 = (x+11)(x-11)$

2. $9y^2 - 169x^2 = (3y-13x)(3y+13x)$

3. $x^4 - 16 = (x-2)(x+2)(x^2+4)$

Be careful!

Sum and Difference of Cubes:

$$a^3 + b^3 = (a+b)\left(a^2 - ab + b^2\right)$$

$$a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right)$$

Write each monomial as a cube and apply either of the rules.

Rewrite as cubes

Example : $\boldsymbol{x^3 + 64} = (x^3 + 4^3)$

Apply the rule for sum of cubes:

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$= (x+4)(x^2 - x \cdot 4 + 4^2)$$

$$= \boxed{(x+4)(x^2 - 4x + 16)}$$

Rewrite as cubes

$$Ex: \quad 8y^3 - 125 = ((2y)^3 - 5^3)$$

Apply the rule for difference of cubes:

$$a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right)$$

$$= (2y-5)\left((2y)^2 + 2y \cdot 5 + (5)^2\right)$$

$$= \boxed{(2y-5)\left(4y^2 + 10y + 25\right)}$$

Factoring Method #3

Factoring a *trinomial* in the form:

$$ax^2 + bx + c$$

Factoring a trinomial: $ax^2 + bx + c$

1. Write two sets of parenthesis, ( )( ). These will be the *factors* of the trinomial.

2. Product of first terms of both binomials must equal first term of the trinomial. (ax^2)

Next

Factoring a trinomial: $ax^2 + bx + c$

3. The product of last terms of both binomials must equal last term of the trinomial (***c***).

4. Think of the FOIL method of multiplying binomials, the sum of the outer and the *inner* products must equal the middle term (***bx***).

$$\textit{Example}: \quad x^2 - 6x + 8$$

$(x \quad)(x \quad) \rightarrow x \cdot x = x^2$

$(x \quad -2)(x \quad -4) \rightarrow$ O + I = *bx* ?

Factors of +8:

1 & 8	→	$1x + 8x = 9x$
2 & 4	→	$2x + 4x = 6x$
-1 & -8	→	$-1x - 8x = -9x$
-2 & -4	→	$-2x - 4x = -6x$

$$x^2 - 6x + 8 = \boxed{(x-2)(x-4)}$$

Check your answer by using FOIL

F O I L

$$(x-2)(x-4) = x^2 - 4x - 2x + 8$$

$$= x^2 - 6x + 8$$

Lets do another example:

$$6x^2 - 12x - 18$$

Don't Forget Method #1.

Always check for GCF before you do anything else.

$$6(x^2 - 2x - 3)$$ *Find a GCF*

$$6(x - 3)(x + 1)$$ *Factor trinomial*

When $a > 1$ and $c < 1$, there may be more combinations to try!

Example : $6x^2 + 13x - 5$

Step 1:

Find the factors of $6x^2$:

$3x \cdot 2x$

$6x \cdot x$

Example : $6x^2 + 13x - 5$

Step 2: Find the factors of -5:

5 g-1

-5 g 1

(-1 g 5

1 g -5)

Order can make a difference!

$$\textit{Example}: \quad 6x^2 + 13x - 5$$

Step 3: Place the factors inside the parenthesis until O + I = ***bx***.

Try: $(6x - 1)(x + 5)$

F	O	I	L

$$6x^2 + 30x - x - 5$$

O + I = 30 x - x = 29x

This doesn't work!!

$$Example: \quad 6x^2 + 13x - 5$$

Switch the order of the second terms and try again.

$$(6x+5)(x-1)$$

F O I L

$$6x^2 - 6x + 5x - 5$$

O + I = $-6x + 5x = -x$

This doesn't work!!

Try another combination:

Switch to 3*x* and 2*x*

$$(3x - 1)(2x + 5)$$

F O I L

$$6x^2 + 15x - 2x - 5$$

O+I = $15x - 2x = 13x$ **IT WORKS!!**

$$6x^2 + 13x - 5 = \boxed{(3x - 1)(2x + 5)}$$

Factoring Technique #3

continued

Factoring a ***perfect square trinomial*** in the form:

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

Perfect Square Trinomials can be factored just like other trinomials (guess and check), but **if** you recognize the perfect squares pattern, follow the formula!

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

$$Ex: \quad x^2 + 8x + 16$$

$$(x)^2 \qquad\qquad (4)^2$$

a $\qquad$ b

Does the middle term fit the pattern, $2ab$?

$$2 \cdot x \cdot 4 = 8x$$

Yes, the factors are $(a + b)^2$:

$$x^2 + 8x + 16 = \boxed{(x + 4)^2}$$

$$Ex: \quad 4x^2 - 12x + 9$$

$$(2x)^2 \qquad\qquad (3)^2$$

a b

Does the middle term fit the pattern, $2ab$?

$$2 \cdot 2x \cdot 3 = 12x$$

Yes, the factors are $(a - b)^2$:

$$4x^2 - 12x + 9 = \boxed{(2x - 3)^2}$$

Factoring Technique #4

Factoring By Grouping
for polynomials
with 4 or more terms

Factoring By Grouping

1. Group the first set of terms and last set of terms with parentheses.
2. Factor out the GCF from each group so that both sets of parentheses contain the same factors.
3. Factor out the GCF again (the GCF is the factor from step 2).

Example 1: $b^3 - 3b^2 + 4b - 12$

Step 1: Group

$$= \left(b^3 - 3b^2\right) + (4b - 12)$$

Step 2: Factor out GCF from each group

$$= b^2(b-3) + 4(b-3)$$

Step 3: Factor out GCF again

$$= \boxed{(b-3)\left(b^2+4\right)}$$

Example 2: $2\boldsymbol{x}^3 - 16\boldsymbol{x}^2 - 8\boldsymbol{x} + 64$

$$= 2\left(x^3 - 8x^2 - 4x + 32\right)$$

$$= 2\left(\left(x^3 - 8x^2\right) + (-4x + 32)\right)$$

$$= 2\left(x^2(x - 8) + -4(x - 8)\right)$$

$$= 2\left((x - 8)\left(x^2 - 4\right)\right)$$

$$= \boxed{2\left((x - 8)(x - 2)(x + 2)\right)}$$

Try these on your own:

1. $x^2 - 5x - 6$

2. $3x^2 + 11x - 20$

3. $x^3 + 216$

4. $8x^3 - 8$

5. $3x^3 - 6x^2 - 24x$

Answers:

1. $(x-6)(x+1)$
2. $(3x-4)(x+5)$
3. $(x+6)(x^2-6x+36)$
4. $8(x-1)(x^2+x+1)$
5. $3x(x-4)(x+2)$