Solving Exponential and Logarithmic Equations

Same Base

- Solve: $4^{x-2} = 64^x$
- $4^{x-2} = (4^3)^x$
- $4^{x-2} = 4^{3x}$
- $x–2 = 3x$
- $-2 = 2x$
- $-1 = x$

$64 = 4^3$

If $b^M = b^N$, then $M = N$

If the bases are already =, just solve the exponents

You Do

- Solve $27^{x+3} = 9^{x-1}$

$$\left(3^3\right)^{x+3} = \left(3^2\right)^{x-1}$$

$$3^{3x+9} = 3^{2x-2}$$

$$3x + 9 = 2x - 2$$

$$x + 9 = -2$$

$$x = -11$$

Review – Change Logs to Exponents

- $\log_3 x = 2$ — $3^2 = x$, $x = 9$
- $\log_x 16 = 2$ — $x^2 = 16$, $x = 4$
- $\log 1000 = x$ — $10^x = 1000$, $x = 3$

Using Properties to Solve Logarithmic Equations

- If the exponent is a variable, then take the natural log of both sides of the equation and use the appropriate property.
- Then solve for the variable.

Example: Solving

- $2^x = 7$ → problem
- $\ln 2^x = \ln 7$ → take ln both sides
- $x \ln 2 = \ln 7$ → power rule
- $x = \frac{\ln 7}{\ln 2}$ → divide to solve for x
- $x = 2.807$

Example: Solving

- $e^x = 72$ → problem
- $\ln e^x = \ln 72$ → take ln both sides
- $x \ln e = \ln 72$ → power rule
- $x = 4.277$ → solution: because

ln e = ?

You Do: Solving

- $2e^x + 8 = 20$ → problem
- $2e^x = 12$ → subtract 8
- $e^x = 6$ → divide by 2
- $\ln e^x = \ln 6$ → take ln both sides
- $x \ln e = 1.792$ → power rule

$x = 1.792$ (remember: $\ln e = 1$)

Example

- Solve $5^{x-2} = 4^{2x+3}$
- $\ln 5^{x-2} = \ln 4^{2x+3}$
- $(x-2)\ln 5 = (2x+3)\ln 4$
- The book wants you to distribute…
- Instead, divide by ln4
- $(x-2)1.1609 = 2x+3$
- $1.1609x-2.3219 = 2x+3$
- $x \approx 6.3424$

Solving by Rewriting as an Exponential

- Solve $\log_4(x+3) = 2$
- $4^2 = x+3$
- $16 = x+3$
- $13 = x$

You Do

- Solve 3ln(2x) = 12
- ln(2x) = 4
- Realize that our base is e, so
- $e^4 = 2x$
- $x \approx 27.299$

- You always need to check your answers because sometimes they don’t work!

Using Properties to Solve Logarithmic Equations

- 1. Condense both sides first (if necessary).
- 2. If the bases are the same on both sides, you can cancel the logs on both sides.
- 3. Solve the simple equation

Example: Solve for x

- $\log_3 6 = \log_3 3 + \log_3 x$ → problem
- $\log_3 6 = \log_3 3x$ → condense
- $6 = 3x$ → drop logs
- $2 = x$ → solution

You Do: Solve for x

- log 16 = x log 2 → problem
- log 16 = log 2^x → condense
- 16 = 2x → drop logs
- x = 4 → solution

You Do: Solve for x

- $\frac{1}{3}\log_4 x = \log_4 4$ → problem
- $\log_4 x^{\frac{1}{3}} = \log_4 4$ → condense
- $x^{\frac{1}{3}} = 4$ → drop logs
- $\left(x^{\frac{1}{3}}\right)^3 = 4^3$ → cube each side
- $X = 64$ → solution

Example

- $7x\log_2 5 = 3x\log_2 5 + \frac{1}{2}\log_2 25$
- $\log_2 5^{7x} = \log_2 5^{3x} + \log_2 25^{\frac{1}{2}}$
- $\log_2 5^{7x} = \log_2 5^{3x} + \log_2 5^1$
- $7x = 3x + 1$
- $4x = 1$

$$x = \frac{1}{4}$$

You Do

- Solve: $\log_7 7 + \log_7 2 = \log_7 x + \log_7 (5x - 3)$

You Do Answer

- Solve: $\log_7 7 + \log_7 2 = \log_7 x + \log_7(5x - 3)$
- $\log_7 14 = \log_7 x(5x - 3)$
- $14 = 5x^2 - 3x$
- $0 = 5x^2 - 3x - 14$
- $0 = (5x + 7)(x - 2)$
- $x = \frac{-7}{5}, 2$

Do both answers work?

NO!!

Final Example

- How long will it take for $25,000 to grow to $500,000 at 9% annual interest compounded monthly?

$$A(t) = A_0\left(1 + \frac{r}{n}\right)^{nt}$$

Example

$$A(t) = A_0\left(1 + \frac{r}{n}\right)^{nt}$$

$$500{,}000 = 25{,}000\left(1 + \frac{0.09}{12}\right)^{12t}$$

$$20 = (1.0075)^{12t}$$

$$12t\ln(1.0075) = \ln 20$$

$$t = \frac{\ln 20}{12\ln 1.0075}$$

$$t \approx 33.4$$