THE

ELLIPSE

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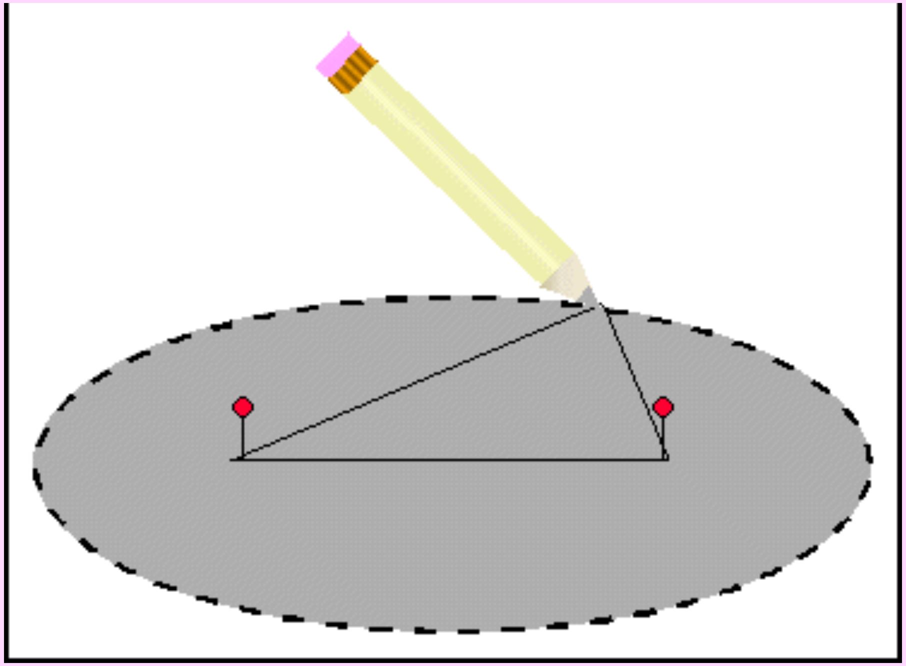
An ellipse is the collection of points in the plane the sum of whose distances from two fixed points, called the foci, is a constant.

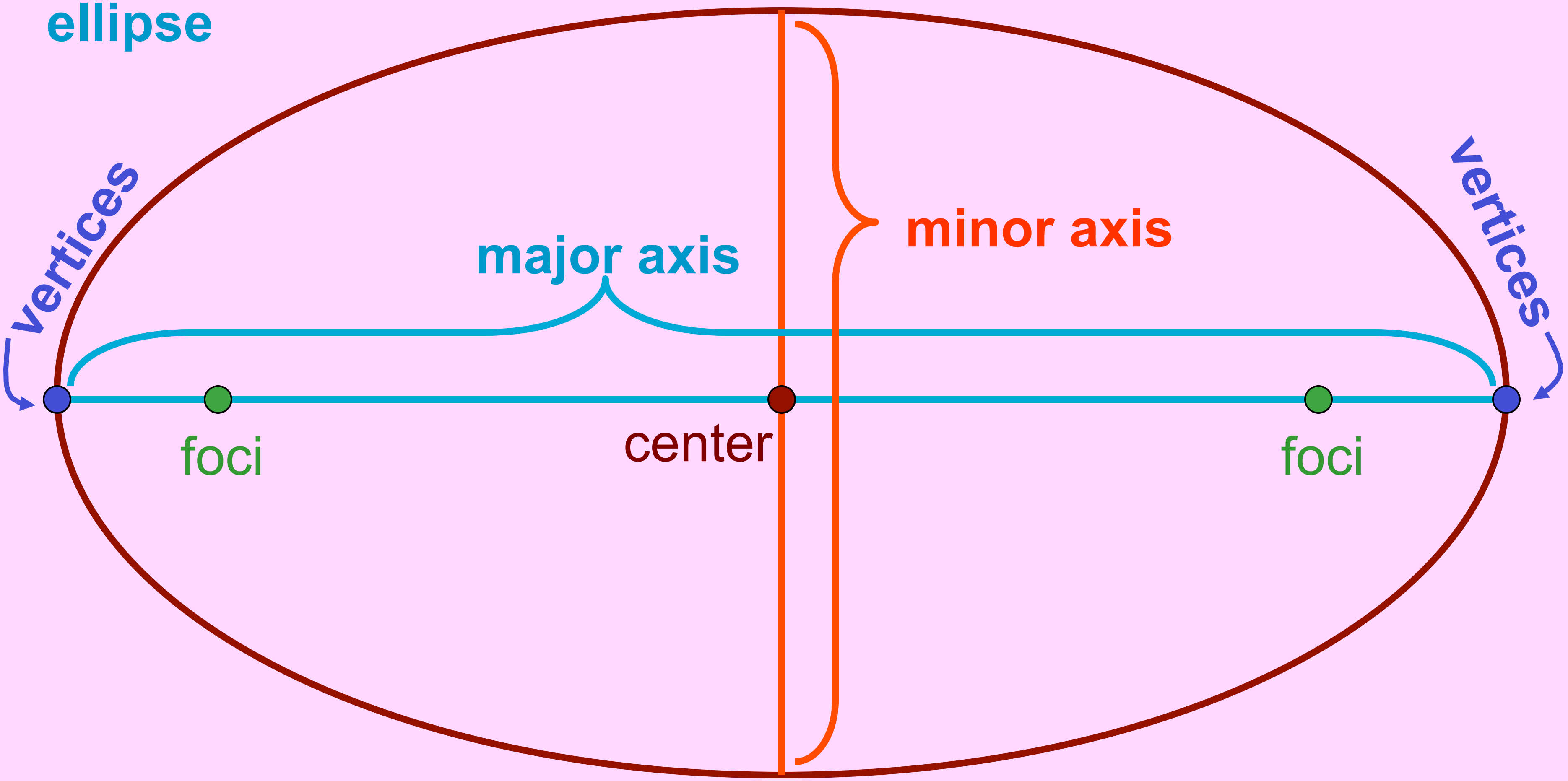
You can draw an ellipse by taking two push pins in cardboard with a piece of string attached as shown:



The place where each pin is is a focus (the plural of which is foci). The sum of the distances from the ellipse to these points stays the same because it is the length of the string.

PARTS OF AN ELLIPSE

The major axis is in the direction of the longest part of the ellipse

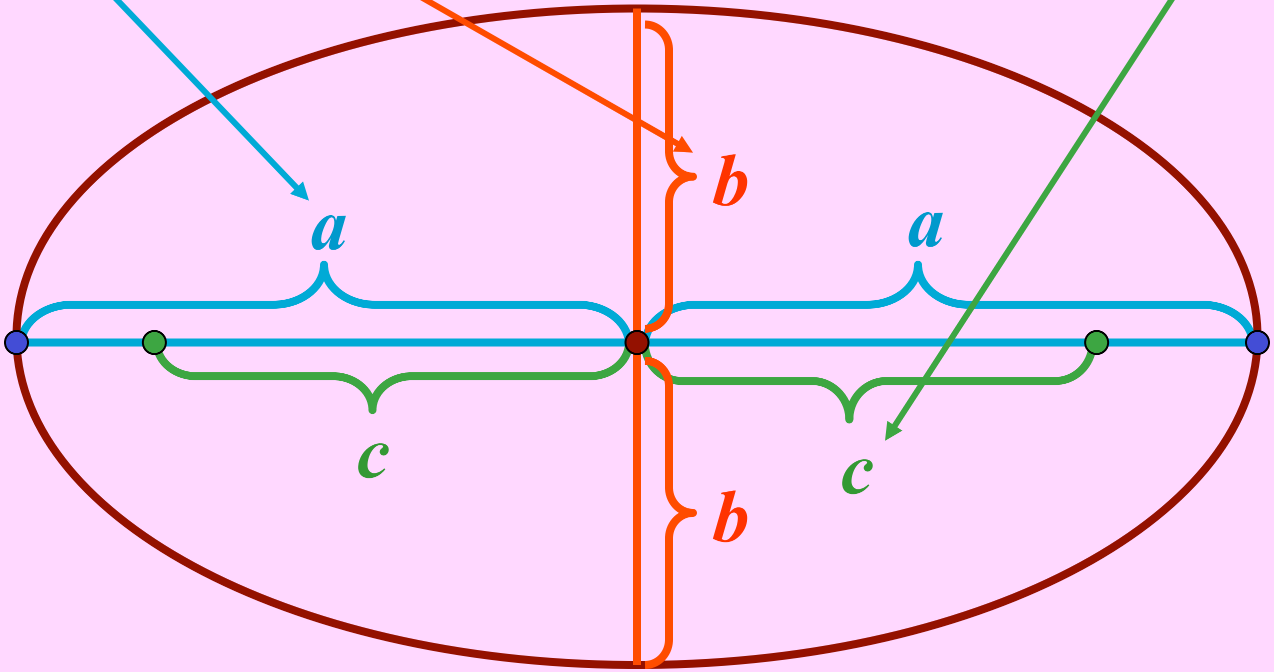


The vertices are at the ends of the major axis

The equation for an ellipse can be derived by using the definition and the distance formula. It is derived in your book on pages 744-45. The resulting equation is:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \text{ where } a > b > 0 \text{ and } b^2 = a^2 - c^2$$

The values of ***a***, ***b***, and ***c*** tell us about the size of our ellipse.



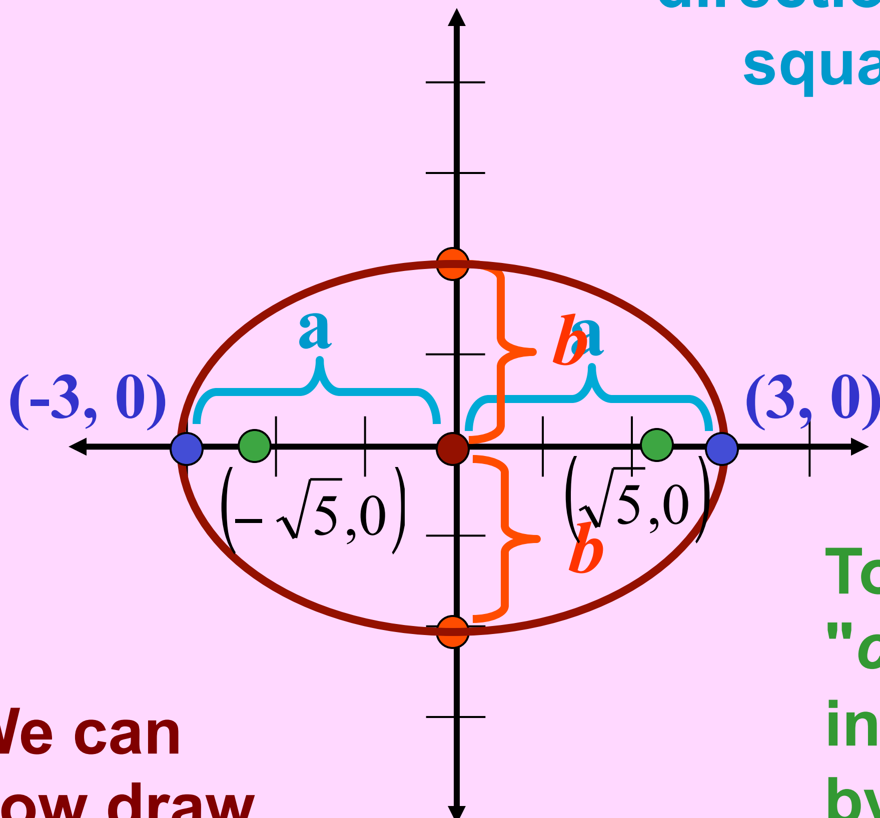
Find the vertices and foci and graph the ellipse:

$$\frac{x^2}{9}+\frac{y^2}{4}=1$$

The ends of this axis are the vertices

From the center the ends of major axis are "*a*" each direction. "*a*" is the square root of this value

From the center the ends of minor axis are "*b*" each direction. "*b*" is the square root of this value



To find the foci, they are "*c*" away from the center in each direction. Find "*c*" by the equation:

$$c^2=a^2-b^2$$

We can now draw the ellipse

$$c^2=9-4=5 \qquad c=\sqrt{5}\approx 2.2$$

The center of the ellipse may be transformed from the origin. The equation would then be:

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1 \qquad \frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$

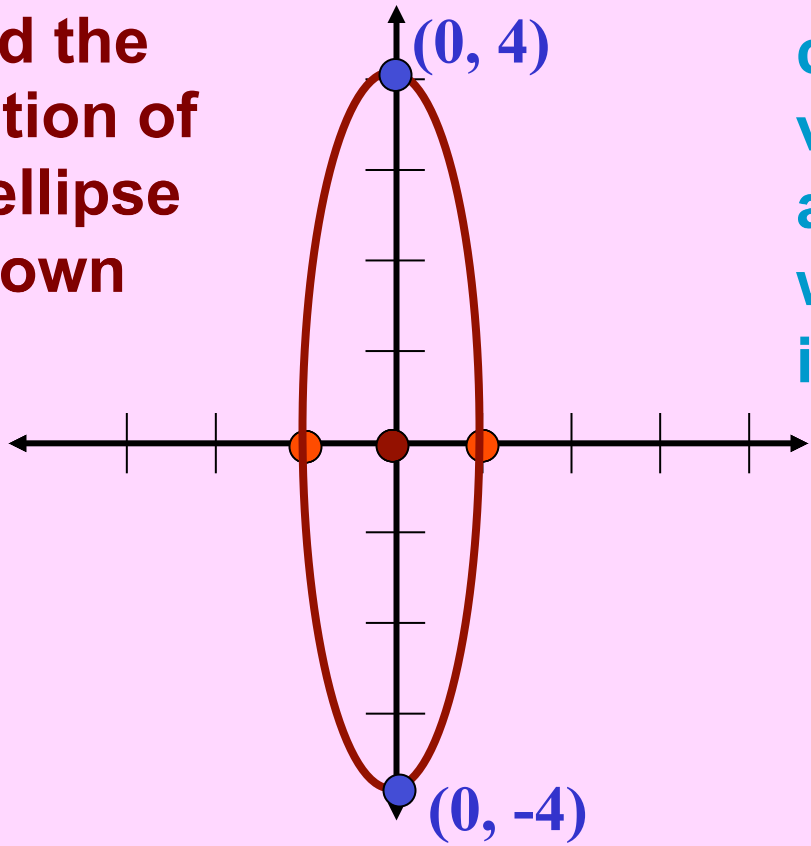
If an ellipse is not in this standard form, you must do algebraic manipulation to get it looking like this. The right hand side must always be a 1.

An ellipse can have a vertical major axis. that case the a^2 is under the y^2

$$\frac{x^2}{1} + \frac{y^2}{16} = 1$$

You can tell which value is *a* because a^2 is always greater than b^2

Find the equation of the ellipse shown



From the center, the vertices are 4 each way so "*a*" is 4.

From the center the ends of minor axis are 1 each direction so "*b*" is 1

To find the foci, they are "*c*" away from the center in each direction along the major axis. Find "*c*" by the equation:

$$c^2 = a^2 - b^2$$

$$c^2 = 16 - 1 = 15 \qquad c = \sqrt{15} \approx 3.9$$

Find the center, foci, vertices and graph the ellipse

complete the square on the *x* terms and then on the *y* terms

$$4x^2 + 3y^2 + 8x - 6y = 5$$

$$4\left(x^2 + 2x + \underline{1}\right) + 3\left(y^2 - 2y + \underline{1}\right) = 5 + 4\left(\underline{1}\right) + 3\left(\underline{1}\right)$$

Here we grouped the *x* terms and factored out a 4 and grouped the y terms and factored out a 3. Since the number you add to complete the square is in parenthesis with a number out in front, that is what you need to add to the other side to keep things equal.

$$\frac{4(x+1)^2}{12} + \frac{3(y-1)^2}{12} = \frac{12}{12}$$

The right hand side must be a 1 so divide all terms by 12

$$\frac{(x+1)^2}{3} + \frac{(y-1)^2}{4} = 1$$

This is now in standard form and we are ready to find what we need and graph (next screen)

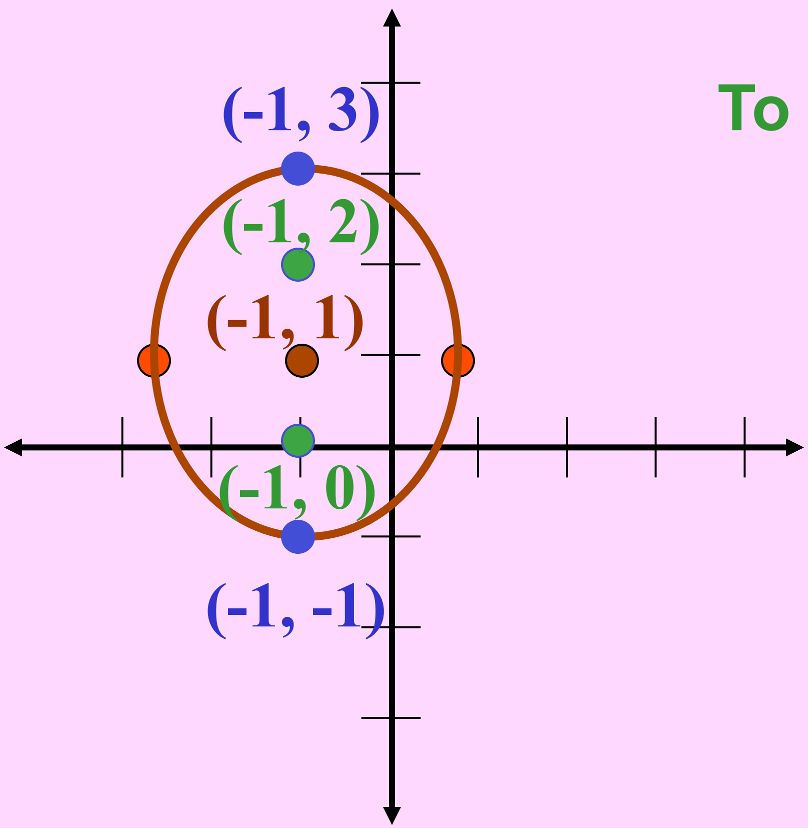
this is b^2 so b = square root of 3

$$\frac{(x+1)^2}{3} + \frac{(y-1)^2}{4} = 1$$

this is a^2 so $a = 2$

The center is at (h, k). In this case (-1, 1).

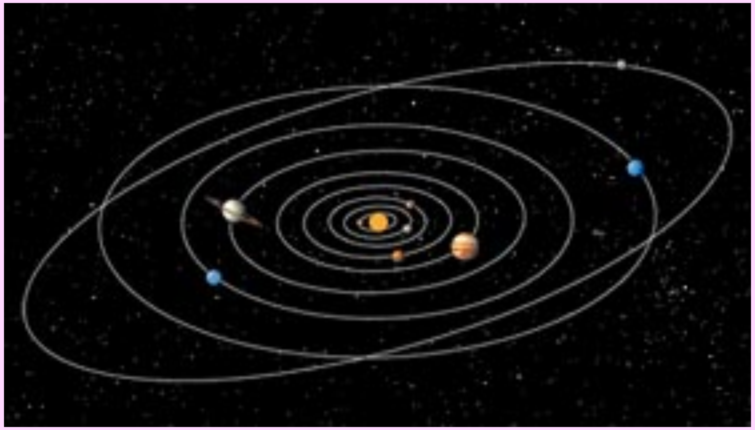
a **= 2 so the vertices (ends of the major axis) are 2 each way from the center. Since the largest number is under the *y*, we move two each way in the *y* direction.**



To find foci: $c^2 = a^2 - b^2$

$$c^2 = 4 - 3 = 1 \qquad c = 1$$

So foci are 1 away from the center in each direction along the major axis



There are many applications of ellipses.

A particularly interesting one is the whispering gallery. The ceiling is elliptical and a person stands at one focus of the ellipse and can whisper and be heard by another person standing at the other focus because all of the sound waves that reach the ceiling from one focus are reflected to the other focus.