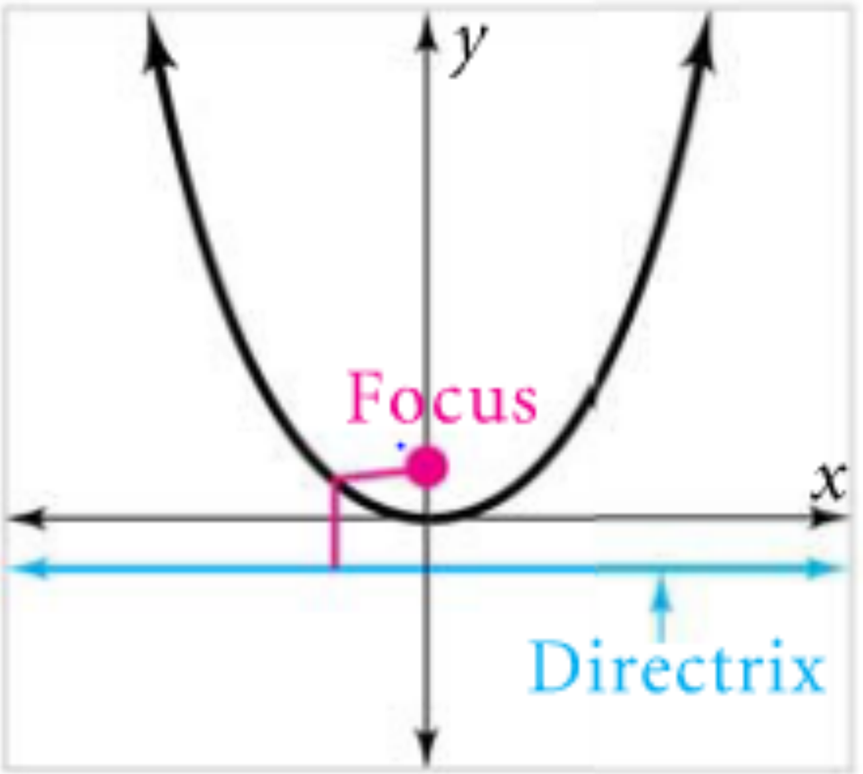
Conics

Parabola

- Parabola: the set of all points in a plane that are equidistance from a fixed line and a fixed point not on the line.
- Focus of a Parabola: the fixed point
- Directrix: Fixed line

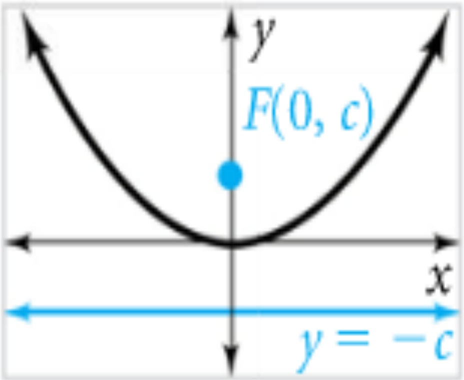
- The line through the focus and perpendicular to the directrix is the axis of symmetry



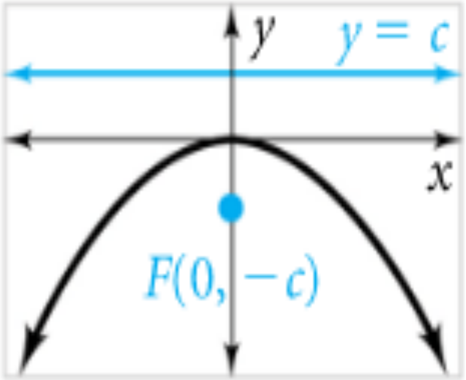
If c is the distance from the vertex to the focus of a parabola, then $|a| = \frac{1}{4c}$.

Consider any parabola with equation $y = ax^2$ and vertex at the origin.

If $a > 0$, then
- the parabola opens upward
- the focus is at $(0, c)$
- the directrix is $y = -c$



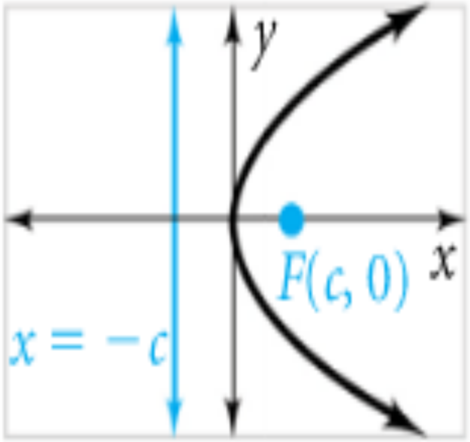
If $a < 0$, then
- the parabola opens downward
- the focus is at $(0, -c)$
- the directrix is at $y = c$



Consider any parabola with equation $x = ay^2$.

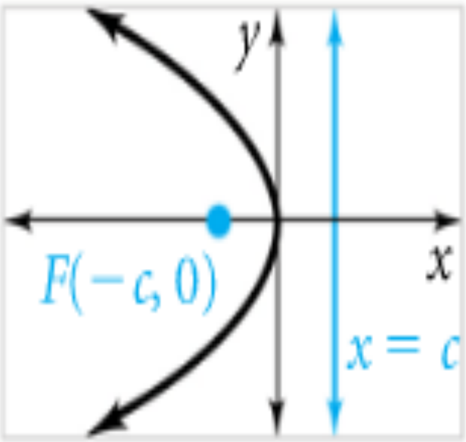
If $a > 0$, then

- the parabola opens to the right
- the focus is at $(c, 0)$
- the directrix is at $x = -c$



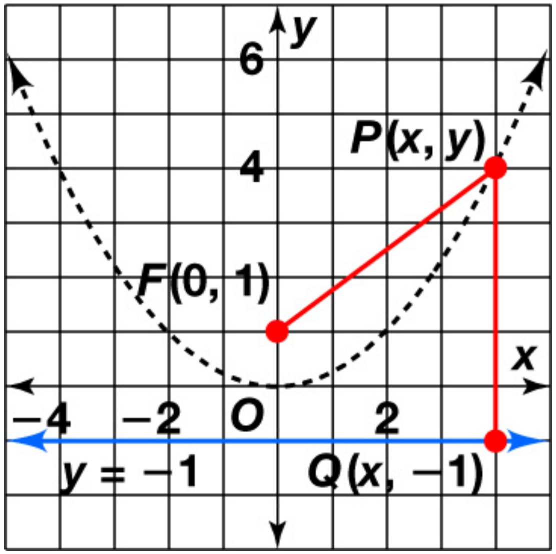
If $a < 0$, then

- the parabola opens to the left
- the focus is at $(-c, 0)$
- the directrix is at $x = c$



Write an equation for a graph that is the set of all points in the plane that are equidistant from point $F(0, 1)$ and the line $y = -1$.

You need to find all points $P(x, y)$ such that FP and the distance from P to the given line are equal.



OBJECTIVE 1

1 EXAMPLE (continued)

$$FP = PQ$$

$$\sqrt{(x - 0)^2 + (y - 1)^2} = \sqrt{(x - x)^2 + (y - (-1))^2}$$

$$x^2 + (y - 1)^2 = 0^2 + (y + 1)^2$$

$$x^2 + y^2 - 2y + 1 = y^2 + 2y + 1$$

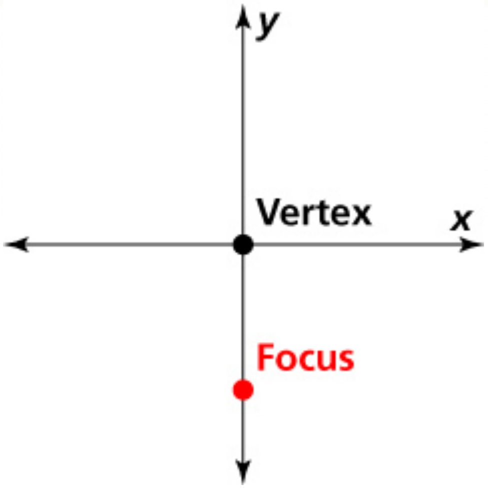
$$x^2 = 4y$$

$$y = \frac{1}{4}x^2$$

An equation for a graph that is the set of all points in the plane that are equidistant from the point F (0, 1) and the line $y = -1$ is $y = \frac{1}{4}x^2$.

OBJECTIVE 1

2 EXAMPLE Write an equation for a parabola with a vertex at the origin and a focus at (0, –7).



Step 1: Determine the orientation of the parabola.

Make a sketch.

Since the focus is located below the vertex, the parabola must open downward. Use $y = ax^2$.

Step 2: Find *a*.

OBJECTIVE 1

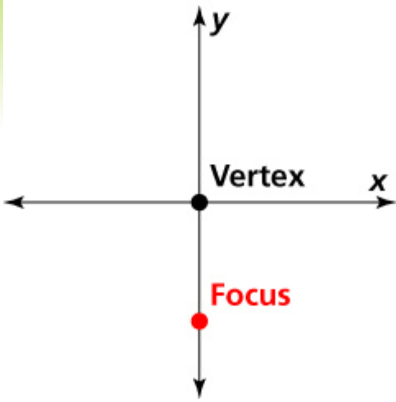
2 EXAMPLE (continued)

$$|a| = \frac{1}{4c}$$

$$= \frac{1}{4(7)}$$

Since the focus is a distance of 7 units from the vertex, $c = 7$.

$$= \frac{1}{28}$$



Since the parabola opens downward, a is negative.

So $a = -\frac{1}{28}$.

An equation for the parabola is $y = -\frac{1}{28}x^2$.

OBJECTIVE 1

3 EXAMPLE A parabolic mirror has a focus that is located 4 in. from the vertex of the mirror. Write an equation of the parabola that models the cross section of the mirror.

The distance from the vertex to the focus is 4 in., so $c = 4$. Find the value of a.

$$a = \frac{1}{4c}$$

$$= \frac{1}{4(4)}$$

$$= \frac{1}{16}$$

Since the parabola opens upward, a is positive.

The equation of the parabola is $y = \frac{1}{16}x^2$.

Identify the focus and directrix of the graph of the equation $x = -\frac{1}{8}y^2$.

The parabola is of the form $x = ay^2$, so the vertex is at the origin and the parabola has a horizontal axis of symmetry. Since $a < 0$, the parabola opens to the left.

$$|a| = \frac{1}{4c}$$

$$\left|-\frac{1}{8}\right| = \frac{1}{4c}$$

$$4c = 8$$

$$c = 2$$

The focus is at (–2, 0). The equation of the directrix is $x = 2$.

OBJECTIVE 2

5 EXAMPLE Identify the vertex, the focus, and the directrix of the graph of the equation $x^2 + 4x + 8y - 4 = 0$. Then graph the parabola.

$x^2 + 4x + 8y - 4 = 0$

$8y = -x^2 - 4x + 4$ Solve for y, since y is the only term.

$8y = -(x^2 + 4x + 4) + 4 + 4$ Complete the square in x.

$y = -\frac{1}{8}(x + 2)^2 + 1$ vertex form

The parabola is of the form $y = a(x - h)^2 + k$, so the vertex is at (–2, 1) and the parabola has a vertical axis of symmetry. Since $a < 0$, the parabola opens downward.

OBJECTIVE 2

5 EXAMPLE (continued)

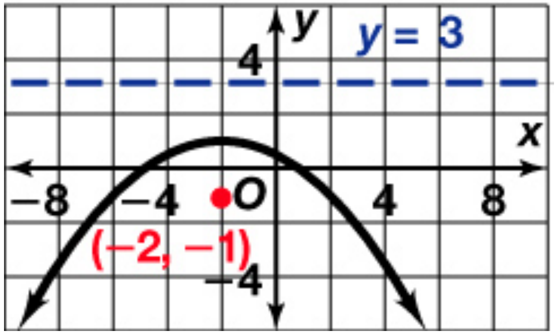
$$|\,a\,| = \frac{1}{4c}$$

$$\left|\,-\frac{1}{8}\,\right| = \frac{1}{4c}$$ Substitute $-\frac{1}{8}$ for *a*.

$$4c = 8$$ Solve for *c*.

$$c = 2$$

The vertex is at (–2, 1) and the focus is at (–2, –1). The equation of the directrix is *y* = 3.



Locate one or more points on the parabola. Select a value for *x* such as –6.
The point on the parabola with an *x*-value of –6 is (–6, –1). Use the symmetric nature of a parabola to find the corresponding point (2, –1).