

Then

- You calculated probabilities. (Lesson PS 4)

Now

- Find probabilities of events given the occurrence of other events.
- Use contingency tables to find conditional probabilities.

Why?

- Alexis is testing a drug that protects people from getting sick. There are two groups; one group gets the experimental drug, while the other group receives a placebo.

After getting the results, Alexis needs to find the probability that a subject's staying healthy was a result of using the experimental drug.

This is an example of a conditional probability.



New Vocabulary

conditional probability
contingency table
two-way table
relative frequency



Tennessee Curriculum Standards

CLE 3103.5.4 Develop an understanding of probability concepts in order to make informed decisions.

✓ **3103.5.13** Apply both theoretical and experimental probability to analyze the likelihood of an event.

SPI 3103.5.8 Apply probability concepts such as conditional probability and independent events to calculate simple probability. Also addresses ✓3103.5.3.

1 Conditional Probability The probability of an event given that another event has already occurred is called **conditional probability**. The conditional probability that event B occurs given that event A has already occurred can be represented by $P(B | A)$. This is read *the probability of B given A*.

KeyConcept Conditional Probability

Given that A and B are dependent events, the conditional probability of an event B , given that an event A has already occurred, is defined as

$$P(B | A) = \frac{P(A \text{ and } B)}{P(A)}, \text{ where } P(A) \neq 0.$$



Example 1 Conditional Probability

Carolina rolls a six-sided die. What is the probability that she has rolled a 3 given that she has rolled an odd number?

There are 6 possible results of rolling a six-sided die.

Let event A be that she rolled an odd number.

Let event B be that she rolled a 3.

$$P(A) = \frac{1}{2}$$

Three of the six outcomes are odd numbers.

$$P(A \text{ and } B) = \frac{1}{6}$$

One of the six outcomes is 3 and odd.

$$P(B | A) = \frac{P(A \text{ and } B)}{P(A)}$$

Probability of B given A

$$= \frac{1}{6} \div \frac{1}{2} \text{ or } \frac{1}{3}$$

$$P(A) = \frac{1}{2} \text{ and } P(A \text{ and } B) = \frac{1}{6}$$

The probability of rolling a 3 given that the roll is odd is $\frac{1}{3}$.

Guided Practice

- Chen draws a card from a standard deck of 52 cards. Find the probability that he drew a king given that he drew a king, a queen, or a jack.



2 Contingency Tables A **contingency table** or **two-way table** records data in which 2 different possible situations result in different possible outcomes. Each value represents the **relative frequency** of an outcome. These tables can be used to find conditional probabilities.



Real-World Example 2 Contingency Tables

MEDICINE Find the probability that a test subject stayed healthy, given that he or she used an experimental drug.

Condition	Number of Subjects	
	Using Drug (D)	Using Placebo (P)
sick (S)	1600	1200
healthy (H)	800	400

There is a total of $1600 + 800 + 1200 + 400$ or 4000 people in the study. We need to find the probability of H given that D occurs.

$$\begin{aligned}
 P(H | D) &= \frac{P(H \text{ and } D)}{P(D)} && \text{Conditional Probability Formula} \\
 &= \frac{800}{4000} \div \frac{2400}{4000} && P(H \text{ and } D) = \frac{800}{4000} \text{ and } P(D) = \frac{1600 + 800}{4000} \\
 &= \frac{800}{2400} \text{ or } \frac{1}{3} && \text{Simplify.}
 \end{aligned}$$

The probability that a subject stayed healthy given that he or she used the drug is $\frac{1}{3}$.

StudyTip

Independent Events

If A and B are independent events, then $P(B | A) = P(B)$.

GuidedPractice

- Find the probability that a test subject remained healthy, if the placebo was used.

Contingency tables can be used to represent any number of possible situations.

SPI 3103.5.8



Test Example 3

The table below shows the number of students who are varsity athletes. Find the probability that a student is a varsity athlete given he or she is a junior.

- A 19.8% C 11.5%
 B 13.0% D 16.6%

Class	Freshman	Sophomore	Junior	Senior
varsity	7	22	36	51
nonvarsity	269	262	276	257

Read the Test Item

We need to find the probability that a student is a varsity athlete given that he or she is a junior. There is a total of 1180 students.

Solve the Test Item

$$\begin{aligned}
 P(V | J) &= \frac{P(V \text{ and } J)}{P(J)} && \text{Conditional Probability Formula} \\
 &= \frac{36}{1180} \div \frac{312}{1180} && P(V \text{ and } J) = \frac{36}{1180} \text{ and } P(J) = \frac{36 + 276}{1180} \\
 &\approx 11.5\% && \text{The correct answer is C.}
 \end{aligned}$$

GuidedPractice

- Find the probability that a student plays varsity given that he or she is a freshman.
 F 2.6% G 2.5% H 8.4% J 7.7%



Math HistoryLink

Christian Huygens

(1629–1695) This Dutchman was the first to discuss games of chance. “Although in a pure game of chance the results are uncertain, the chance that one player has to win or to lose depends on a determined value.” This became known as the *expected value*.





Example 1 A bag contains 8 blue marbles, 6 red marbles, 10 yellow marbles, 6 white marbles, and 5 green marbles. A marble is chosen at random. Find each probability.

1. The marble is green, given that it is not blue.
2. The marble is red, given that it is not green.
3. The marble is yellow, given that it is not red or blue.
4. The marble is green or white, given that it is not red.

Example 2 5. **DRIVING TESTS** The table shows how students in Mr. Diaz’s class fared on their first driving test. Some took a class to prepare, while others did not.

	Class	No Class
Passed	64	48
Failed	18	32

- a. Find the probability that Paige passed, given that she took the class.
- b. Find the probability that Elizabeth failed, given that she did not take the class.
- c. Find the probability that Terrence did not take the class, given that he passed.

Example 3 6. **MULTIPLE CHOICE** The number of students who have attended a football game at North Coast High School is listed below. Find the probability that a student who has attended a game is a junior or a senior.

Class	Freshman	Sophomore	Junior	Senior
attended	48	90	224	254
not attended	182	141	36	8

- A 48.6% B 77.6% C 86.2% D 91.6%

Practice and Problem Solving

Extra Practice begins on page 947.

Example 1 A tip jar contains 7 pennies, 15 nickels, 25 dimes, and 32 quarters. A coin is chosen at random. Find each probability.

7. The coin is a nickel, given that it is silver.
8. The coin is a quarter, given that it is not a dime.
9. The coin is a penny, given that it is not a quarter.
10. The coin is a dime or a quarter, given that it is silver.

Example 2 11. **SCHOOL CLUBS** King High School tallied the number of males and females that were members of at least one after school club. Find each probability.

	Clubs	No Clubs
Male	156	242
Female	312	108

- a. A student is a member of a club given that he is male.
- b. A student is not a member of a club given that she is female.
- c. A student is a male given that he is not a member of a club.

Example 3 12. **MULTIPLE CHOICE** Naoko, Keisha, and Joshua compared the music on their MP3 players. Find the probability that a selected song is country given that it is not on Naoko’s player.

Person	Rock	Country	R & B
Naoko	521	316	44
Keisha	119	145	302
Joshua	244	4	182

- F 17.2% G 24.8% H 35.9% J 15.0%



A card is chosen at random from a standard deck. Find each probability. Assume that an ace represents a 1.

13. The card is a ten, given that it is red.
14. The card is a five, given that it is not a face card.
15. The card is a queen, given that it is a face card.
16. The card is greater than 7, given that it is not a face card.

SOFTBALL On average, Paloma gets a single 14% of the time, a double 6% of the time, a triple 1% of the time, a home run 13% of the time, and is walked 3% of the time.

17. What is the probability that Paloma gets a double, given that she is not out?
18. What is the probability that Paloma gets a triple or a home run, given that she is not walked?

19. **COMPUTER GAMES** The table shows a distribution of computer games sold by a company.

Type	P
strategy	0.19
children's	0.12
family	0.08
action	0.25
role playing	0.17
sports	0.16
other	0.03

- a. Find the probability that a game is an action game, given that it is not a sports or role playing game.
 - b. Find the probability that a game is a family game, given that it is not a strategy or action game.
20. **FIRE DRILLS** Marburn High School will have a fire drill at a randomly-chosen time between 8:30 and 3:00. Mr. Woodruff has a test planned for 1:20 to 1:55. If the fire drill is in the afternoon, what is the probability that it will start during the test?
21. **FUNDRAISING** Mercedes and Victoria are trying to raise funds for their charity by calling numbers in the local phone book and asking for donations. They only reach 40% of the people they call. Of the people they reach, 20% promise to donate funds. Of the people who promise to donate, only 25% actually send money. What is the probability that a person who is called will actually contribute?
22. **HONORS CLASS** The probability that a student is in honors, given that he or she is in Mrs. Rollins' class, is $\frac{28}{51}$. The probability that a student is not in Mrs. Rollins' class, given that he or she is not in honors, is $\frac{33}{56}$. If there are 165 students that are neither in Mrs. Rollins' class nor in honors, how many students *are* in Mrs. Rollins' class and in honors?

H.O.T. Problems Use Higher-Order Thinking Skills

23. **CHALLENGE** The probability that a student has a MyRoom page, given that he or she is a freshman, is $\frac{43}{55}$. The probability that a student does not have a MyRoom page, given that he or she is a sophomore, is $\frac{4}{27}$. If there are 82 students, determine the probability that a student is a freshman, given that he or she does have a MyRoom page.
24. **WRITING IN MATH** Explain the difference between conditional probability for dependent events and conditional probability for independent events. Provide examples of each type.
25. **REASONING** Which branches of a tree diagram represent conditional probability? Provide a sample tree diagram and explain your reasoning.
26. **REASONING** If a fair coin is flipped 20 times in a row and comes up heads every single time, what is the probability that it comes up heads on the 21st flip? Explain your reasoning.
27. **OPEN ENDED** Create a contingency table and calculate a conditional probability using the students in your class.

