Problems Solved Chapter 12_Logarithms

Solving Exponential and Logarithmic Equations

Here we will make use of what we have learned about exponentials and logarithms to solve equations.

One-to-One Property of Exponential Functions –				
If $b^n = b^m$	then	n = m		

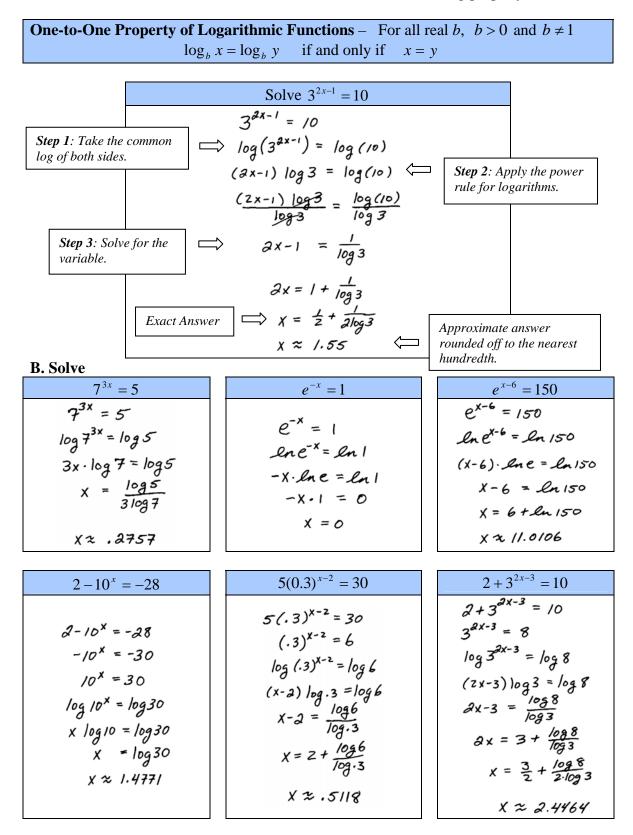
We may make use of the above property if we are able to express both sides of the equation in terms of the same base as shown here.

	Solve $5^{3x-2} = 125^{2x}$	Recall the rule of exponent $(x^n)^m = x^{nm}$
	$5^{3x-2} = 125^{2x}$ $5^{3x-2} = (5^3)^{2x}$	
	5 ^{3x-2} = 5 ^{6x} <	Step 1: Express both sides in terms of the same base.
Step 2: Equate the exponents.] ⇒ 3x-2 = 6x -2 = 3x	
		ep 3 : Solve for the uriable.

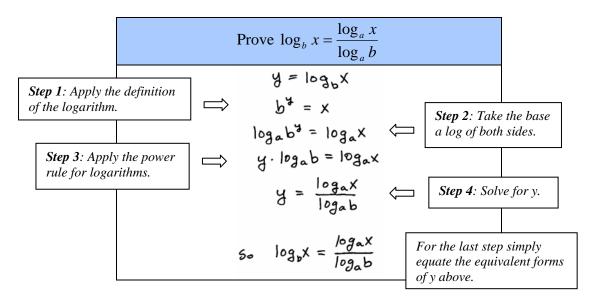
A. Solve

$2^{x} = 8$	$32^{x-2} = 64$	$100^x = 1000^{2x-1}$
2 ^x = 8 2 ^x = 2 ³ x = 3	$32^{x-2} = 64$ (2 ⁵) ^{x-2} = 2 ⁶ 2 ^{5x-10} = 2 ⁶ 5x-10 = 6 5x = 16 x = ¹⁶ / ₅	$\frac{100^{x}}{(10^{2})^{x}} = \frac{1000^{2x-1}}{(10^{3})^{2x-1}}$ $\frac{10^{2x}}{10^{2x}} = \frac{10^{6x-3}}{10^{6x-3}}$ $\frac{10^{6x-3}}{10^{6x-3}}$
	.2. 1	
$81^{-x} = 9^{-x-2}$	$4^{2x-1} = 64$	$36^x = 216^{x-3}$
$81^{-x} = 9^{-x-2}$	4 ^{2x-1} = 64	$36^{\times} = 2/6^{\times -3}$
$(9^2)^{-x} = 9^{-x-2}$	$4^{2x-1} = 4^3$	$(6^2)^{\times} = (6^3)^{\times -3}$
$9^{-2x} = 9^{-x-z}$	2X-/=3	$6^{2x} = 6^{3x-9}$
-Zx = -X - 2	2x = 4	2x = 3x - 9
-x = -2		-x = -9
X = 2	x = 2	x = 9

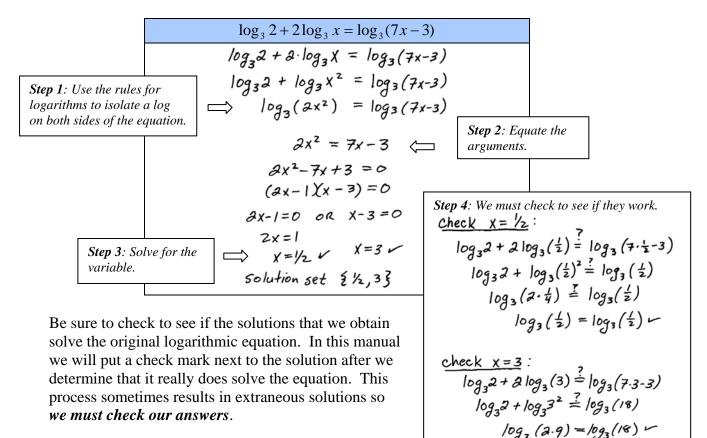
It is not always the case that we will be able to express both sides of an equation in terms of the same base. For this reason we will make use of the following property.



When solving exponential equations and using the above process the rule of thumb is to choose the common logarithm unless the equation involves e. We choose these because there is a button for them on the calculator. But certainly we could use any base we wish; this is the basis for the derivation of the change of base formula.



We can also use the one-to-one property for logarithms to solve logarithmic equations. If we are given an equation with a logarithm of the same base on both sides we may simply equate the arguments.



C. Solve

$\log_2(3x) = \log_2(2x+7)$
$\log_2(3x) = \log_2(2x+7)$
3x = 2x + 7
x = 7 い
Check: $\log_2(3.7) \stackrel{?}{=} \log_2(3.7+7)$
10gg(21) = 10g(21) -
Solution Set {7}

$$\ln(2x+10) = 2\ln 2$$

$$\ln(2x+10) = 2\ln 2$$

$$\ln(2x+10) = 2\ln 2^{n}$$

$$2x+10 = 4^{n}$$

$$2x = -6$$

$$x = -3 - \frac{1}{2}$$

$$Check : x = -3$$

$$\ln(2(-3)+10) \stackrel{?}{=} 2\ln 2$$

$$ln 4 = \ln 2^{2} - \frac{1}{2}$$

$$Solution Set \xi - 33$$

 $\log(x+1) + \log(x-1) = \log 8$

log(x+1) + log(x-1) = log 8

log(x+1)(x-1) = log 8 $log(x^2-1) = log 8$ $x^2-1 = 8$

 $X^2 = 9$

K=-3 OR X=3-

Solution Set \$33

extraneous

$$2\ln x = \ln 25$$

$$2\ln x = \ln 25$$

$$\ln x^{2} = \ln 25$$

$$x^{2} = 25$$

$$x^{2} = 25$$

$$x = -5 \quad or \quad x = 5 - -$$

extraneous
check x = 5: $2\ln 5 = \ln 25$
 $\ln 5^{2} = \ln 25 - -$
check x = -5: $2\ln (-5) = \ln 25$
unochined 5
solution set 5

$$log_{2} 5 + log_{2} x = log_{2}(x+20)$$

$$log_{2} 5 + log_{2} X = log_{2}(x+20)$$

$$log_{2}(5x) = log_{2}(x+20)$$

$$5x = x+20$$

$$4x = 20$$

$$x = 5 - -$$

$$Check: log_{2} 5 + log_{2} 5 = log_{2}(5+20)$$

$$log_{2}(5\cdot5) = log_{2}(25)$$

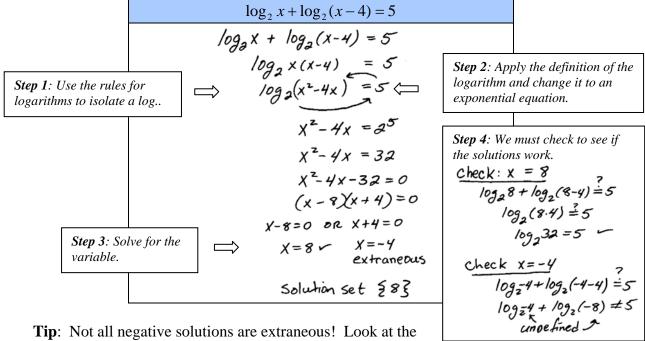
$$log_{2}(25) = log_{2}(25) - -$$

$$Solution Set \ 253$$

 $log_{5} x + log_{5} (x-3) = log_{5} 10$ $log_{5} x + log_{5} (x-3) = log_{5} / 0$ $log_{5} x (x-3) = log_{5} / 0$ $x^{2} - 3x = / 0$ $x^{2} - 3x - 10 = 0$ (x + 2)(x - 5) = 0 x + 2 = 0 or x - 5 = 0 x + 2 = 0 or x - 5 = 0 $x = -2 \quad x = 5 - -$ extraneous $solution 5et \{55\}$

Of course, equations like these are very special. Most of the problems that we will encounter will not have a logarithm on both sides, as with the next set.

If the answer to our logarithmic equation makes the argument of it negative then it is extraneous. This does not preclude negative answers. We must be sure to check all of our solutions.



Tip: Not all negative solutions are extraneous! Look at the following problems and we can see that some have negative answers.

D.	Solve
ν.	BUIVE

 $\log(2x+30) = 1$ $\log_{6}(2x+6) = 2$ $log_{6}(2x+6) = 2$ $\log_{10}(2x+30) = 1$ $2x+6=6^{2}$ 2x+30 = 10'2x + 6 = 362x = -202x = 30x = -10 ~ X=15 -Solution Set 2-103 Solution Set \$153 $\ln(x^2 - 4) = 0$ $\log_{49}(x+1) = \frac{1}{2}$ $\log_{49}(x+1) = \frac{1}{2}$ $ln(x^2-4)=0$ $x^{2} - 4 = e^{0}$ $Y + I = 49^{\frac{1}{2}}$

$$x^{2}-4 = 1$$

$$x^{2} = 5$$

$$x = \pm \sqrt{5}$$
Solution Set $\xi - \sqrt{5}, \sqrt{5}$

$$x = \frac{1}{2} \sqrt{5}$$

The check mark indicates that we actually plugged the answers in to see that they do indeed solve the original. Please do not skip this step, extraneous solutions occur often.

 $\ln x - \ln(x - 1) = 1$ $\log_2(15x-1) - \log_2(x-1) = 4$ lnx - ln(x-1)=1 $\log_2(15 \times -1) - \log_2(X - 1) = 4$ $ln\left(\frac{X}{X-1}\right) = 1$ $\log_{2}\left(\frac{15\chi-1}{\chi-1}\right) = 4$ $\frac{x}{x-1} = e'$ $\frac{15\chi - 1}{\chi - 1} = 2^4$ x = e(x-1)15x-1 = 16(x-1)x = ex - e15x - 1 = 16x - 16x - ex = -e-x = -15x(1-e) = -eX = 15 ~ $X = \frac{-e}{1-e} = \frac{e}{e-1} - \frac{e}{e}$ Solution Set \$153 Solution Set 2 = 3 $\log_5 x + \log_5(2x - 9) = 1$ $\log_4 x + \log_4 (x - 12) = 3$ $\log_{4} X + \log_{4} (X - 12) = 3$ $log_{5} X + log_{5} (\partial X - 9) = l$ 1094 × (x-12) = 3 $log_{5} \times (2x-q) = 1$ $X(X-12) = 4^3$ x(2x-9) = 5' $x^{2} - 12x = 64$ $2x^2 - 9x - 5 = 0$ $x^{2} - 12x - 64 = 0$ (2x+1)(x-5)=0(x + 4)x - 16 = 02x+1=0 OR X-5=0 X+4=0 OR X-16=0 2x=-1 X=5~ X=1/2 extraneous X=16~ X=-4 extraneous Solution Set 253 Solution Set 2163 $\log_{6} x = 1 - \log_{6} (x - 1)$ $2\log(11x-12) + 1 = 3$ $\log_{6} x = 1 - \log_{6}(x-1)$ $log_{6}x + log_{6}(x-1) = 1$ $2 \log(1/x - 12) + 1 = 3$ 2 log(11x-12) =2 109, ×(x-1) = 1 log(11x-12)=1 x(x-1) = 6'1/x - 12 = 10' $x^{2} - x - 4 = 0$ //x = 22 (x+a)(x-3)=0X=2~ X+2=0 OR X-3=0 Solution Set 223 X=3 / X = - L Extraneous

Be sure to check all solutions to logarithmic equations because some might be extraneous!

Solution Set {33