

Lesson 5-6

Complex Numbers

<p>Lesson Objectives</p> <p>1 Identifying and graphing complex numbers</p> <p>2 Adding, subtracting, and multiplying complex numbers</p>	<p>NAEP 2005 Strand: Algebra</p> <p>Topic: Algebraic Representations</p> <p>Local Standards: _____</p>
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Vocabulary and Key Concepts

Square Root of a Negative Real Number

For any positive real number a , $\sqrt{-a} = i\sqrt{a}$.

Example $\sqrt{-4} = i\sqrt{4} = i \cdot 2 = 2i$

Note that $(\sqrt{-4})^2 = (i\sqrt{4})^2 = i^2(\sqrt{4})^2 = -1 \cdot 4 = -4$ (not 4).

Complex Numbers

A complex number can be written in the form $a + bi$, where a and b are real numbers, including 0.

$$\begin{matrix} a & + & bi \\ \uparrow & & \uparrow \\ \text{Real} & & \text{Imaginary} \\ \text{part} & & \text{part} \end{matrix}$$

Complex Numbers							
<p>Real Numbers: $-5, -\sqrt{3}, 0, \sqrt{5}, \frac{8}{3}, 9$</p> <table border="1" style="margin-left: 20px;"> <tr> <td style="padding: 5px;"> <p>Numbers: $-5, 0, \frac{8}{3}, 9$</p> <table border="1" style="margin-left: 20px;"> <tr> <td style="padding: 5px;"> <p>Integers: $-5, 0, 9$</p> <table border="1" style="margin-left: 20px;"> <tr> <td style="padding: 5px;"> <p>Whole Numbers: $0, 9$</p> <table border="1" style="margin-left: 20px;"> <tr> <td style="padding: 5px;"> <p>Natural Numbers: 9</p> </td> </tr> </table> </td> </tr> </table> </td> </tr> </table> </td> <td style="padding: 5px;"> <p>Irrational Numbers:</p> <p>$-\sqrt{3}$</p> <p>$\sqrt{5}$</p> </td> <td style="padding: 5px;"> <table border="1" style="margin-left: 20px;"> <tr> <td style="padding: 5px;"> <p>Numbers:</p> <p>$-4i$</p> <p>$3 + 2i$</p> <p>$2i\sqrt{2}$</p> </td> </tr> </table> </td> </tr> </table>	<p>Numbers: $-5, 0, \frac{8}{3}, 9$</p> <table border="1" style="margin-left: 20px;"> <tr> <td style="padding: 5px;"> <p>Integers: $-5, 0, 9$</p> <table border="1" style="margin-left: 20px;"> <tr> <td style="padding: 5px;"> <p>Whole Numbers: $0, 9$</p> <table border="1" style="margin-left: 20px;"> <tr> <td style="padding: 5px;"> <p>Natural Numbers: 9</p> </td> </tr> </table> </td> </tr> </table> </td> </tr> </table>	<p>Integers: $-5, 0, 9$</p> <table border="1" style="margin-left: 20px;"> <tr> <td style="padding: 5px;"> <p>Whole Numbers: $0, 9$</p> <table border="1" style="margin-left: 20px;"> <tr> <td style="padding: 5px;"> <p>Natural Numbers: 9</p> </td> </tr> </table> </td> </tr> </table>	<p>Whole Numbers: $0, 9$</p> <table border="1" style="margin-left: 20px;"> <tr> <td style="padding: 5px;"> <p>Natural Numbers: 9</p> </td> </tr> </table>	<p>Natural Numbers: 9</p>	<p>Irrational Numbers:</p> <p>$-\sqrt{3}$</p> <p>$\sqrt{5}$</p>	<table border="1" style="margin-left: 20px;"> <tr> <td style="padding: 5px;"> <p>Numbers:</p> <p>$-4i$</p> <p>$3 + 2i$</p> <p>$2i\sqrt{2}$</p> </td> </tr> </table>	<p>Numbers:</p> <p>$-4i$</p> <p>$3 + 2i$</p> <p>$2i\sqrt{2}$</p>
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The symbol i represents _____

An imaginary number is _____

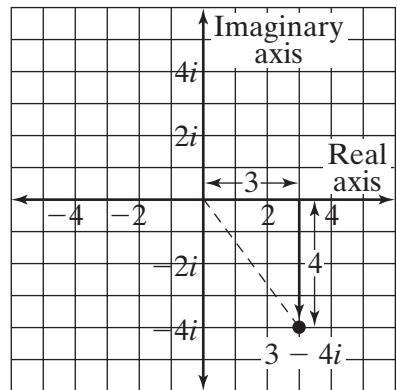
The set of complex numbers is made up of _____

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A complex number plane can be used to _____

The absolute value of a complex number is _____



Examples

- 1 **Simplifying Numbers Using i** Simplify $\sqrt{-54}$ by using the imaginary number i .

$$\begin{aligned} \sqrt{-54} &= \sqrt{\square} \cdot 54 \\ &= \sqrt{\square} \cdot \sqrt{54} \\ &= \square \cdot \sqrt{54} \\ &= \square \cdot \square \sqrt{\square} \\ &= \square \end{aligned}$$

- 2 **Simplifying Imaginary Numbers** Write the complex number $\sqrt{-121} - 7$ in the form $a + bi$.

$$\begin{aligned} \sqrt{-121} - 7 &= \square - 7 && \text{Simplify the radical expression.} \\ &= \square + \square && \text{Write in the form } a + bi. \end{aligned}$$

Quick Check

1. Simplify each number by using the imaginary number i .

a. $\sqrt{-2}$

b. $\sqrt{-12}$

c. $\sqrt{-36}$

2. Write the complex number $\sqrt{-18} + 7$ in the form $a + bi$.

Examples

3 Finding Absolute Value Find each absolute value.

a. $|-7i|$
 $-7i$ is units from the origin on the imaginary axis.
 So $|-7i| = \square$

b. $|10 + 24i|$
 $|10 + 24i| = \sqrt{10^2 + 24^2}$ Use the Pythagorean Theorem to find distance.
 $= \sqrt{\square + \square} = \square$ Simplify.

4 Adding Complex Numbers Simplify the expression $(3 + 6i) - (4 - 8i)$.

$(3 + 6i) - (4 - 8i) = 3 + (-4) + \square + \square$ Use commutative and associative properties.
 $= \square + \square$ Simplify.

5 Multiplying Complex Numbers

a. Find $(3i)(8i)$.
 $(3i)(8i) = \square i^2$ Multiply the real numbers.
 $= \square (\square) = \square$ Substitute -1 for i^2 and multiply.

b. Find $(3 - 7i)(2 - 4i)$.
 $(3 - 7i)(2 - 4i) = 6 - \square - \square + 28i^2$ Multiply the binomials.
 $= 6 - \square + 28(\square)$ Substitute -1 for i^2 .
 $= \square - \square$ Simplify.

Quick Check

3. Find the absolute value of each complex number.

a. $ 6 - 4i $ <input type="text"/>	b. $ -2 + 5i $ <input type="text"/>	c. $ 4i $ <input type="text"/>
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4. Simplify each expression.

a. $(8 + 3i) - (2 + 4i)$ <input type="text"/>	b. $7 - (3 + 2i)$ <input type="text"/>	c. $(4 - 6i) + 3i$ <input type="text"/>
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5. Simplify each expression.

a. $(12i)(7i)$ <input type="text"/>	b. $(6 - 5i)(4 - 3i)$ <input type="text"/>	c. $(4 - 9i)(4 + 3i)$ <input type="text"/>
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Examples

6 Finding Complex Solutions Solve $9x^2 + 54 = 0$.

$$9x^2 + 54 = 0$$

$$9x^2 = \boxed{} \quad \text{Isolate } x^2.$$

$$x^2 = \boxed{}$$

$$x = \pm \boxed{} \quad \text{Find the square root of each side.}$$

7 Output Values Find the first three output values for $f(z) = z^2 - 4i$. Use $z = 0$ as the first input value.

$$f(0) = \boxed{}^2 - 4i$$

$$= \boxed{}$$

$$f(-4i) = (-4i)^2 - 4i$$

$$= \boxed{} - 4i$$

First output becomes second input.
Evaluate for $z = -4i$.

$$f(\boxed{} - \boxed{}) = (\boxed{} - \boxed{})^2 - 4i$$

Second output becomes third input.

Evaluate for $z = \boxed{} - \boxed{}$.

$$= \left[(\boxed{})^2 + (-16)(-4i) + (-16)(-4i) + (\boxed{})^2 \right] - 4i$$

$$= (\boxed{} + \boxed{} - \boxed{}) - 4i$$

$$= \boxed{} + \boxed{}$$

The first three output values are $\boxed{}$, $\boxed{}$, and $\boxed{}$.

Quick Check

6. Solve for x .

a. $3x^2 + 48 = 0$

b. $-5x^2 - 150 = 0$

c. $8x^2 + 2 = 0$

7. Find the first three output values for $f(z) = z^2 - 1 + i$. Use $z = 0$ as the first input value.

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