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Lesson 9-3

Rational Functions and Their Graphs

Lesson Objectives	NAEP 2005 Strand: Algebra
1 Identifying properties of rational functions	Topic: Algebraic Representations
2 Graphing rational functions	Local Standards: ____________________

Key Concepts

Rational Function

A **rational function** $f(x)$ is a function that can be written as $f(x) = \frac{P(x)}{Q(x)}$, where ______ and ______ are polynomial functions. The domain of $f(x)$ is all real numbers except those for which $Q(x) =$ ______.

Vertical Asymptotes

The rational function $f(x) = \frac{P(x)}{Q(x)}$ has a point of discontinuity for each real ______ of $Q(x)$.

If $P(x)$ and $Q(x)$ have no common real zeros, then the graph of $f(x)$ has a ______ asymptote at each real zero of $Q(x)$.

If $P(x)$ and $Q(x)$ have a common real zero a, then there is a ______ in the graph or a vertical asymptote at $x =$ ______.

Horizontal Asymptotes

- The graph of a rational function has at most ______ horizontal asymptote.
- The graph of a rational function has a ______ asymptote at $y = 0$ if the degree of the ______ is ______ than the degree of the ______.
- If the ______ of the numerator and the denominator are equal, then the graph has a ______ asymptote at $y =$ ______, where a is the coefficient of the term of highest ______ in the numerator and b is the coefficient of the term of highest degree in the ______.
- If the degree of the ______ is greater than the degree of the ______, then the graph has ______ horizontal asymptote.

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Example

1 Finding Points of Discontinuity For each rational function, find any points of discontinuity.

a. $y = \dfrac{3}{x^2 - x - 12}$

The function is undefined at values of x for which $x^2 - x - 12 = 0$.

$x^2 - x - 12 = \square$ **Set the denominator equal to** □.

$(x \square 4)(x + \square) = 0$ **Solve by factoring or using the Quadratic Formula.**

$x - 4 = \square$ or $x + 3 = \square$ □ **Property.**

$x = \square$ or $x = \square$ **Solve for *x*.**

There are points of discontinuity at $x = \square$ and $x = \square$.

b. $y = \dfrac{2x}{3x^2 + 4}$

The function is undefined at values of $3x^2 + 4 = 0$.

$3x^2 + 4 = \square$ **Set the denominator equal to zero.**

$x^2 = -\dfrac{\square}{\square}$ **Solve for *x*.**

$$x = \pm\sqrt{\frac{\square}{3}} = \frac{\pm\square}{\sqrt{\square}} = \frac{\pm 2i\sqrt{3}}{3}$$

Since $\frac{\pm 2i\sqrt{3}}{3}$ □ a real number, there is □ real value for x for which the function $y = \frac{2x}{3x^2 + 4}$ is undefined. There is □ point of discontinuity.

Quick Check

1. For each rational function, find any points of discontinuity.

a. $y = \dfrac{1}{x^2 - 16}$

b. $y = \dfrac{x^2 - 1}{x^2 + 3}$

c. $y = \dfrac{x + 1}{x^2 + 2x - 8}$

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Example

2 Finding Vertical Asymptotes Describe the vertical asymptotes and holes for the graph of each rational function.

a. $y = \frac{x - 7}{(x + 1)(x + 5)}$

Since -1 and -5 are the zeros of the ______ and neither is a zero of the numerator, $x = -1$ and $x = -5$ are ______ asymptotes.

b. $y = \frac{(x + 3)x}{x + 3}$

-3 is a zero of ______ the numerator and the denominator. The graph of this function is the same as the graph $y = x$, except it has a hole at $x =$ ______.

Quick Check

2. Describe the vertical asymptotes and holes for the graph of each rational function.

a. $y = \frac{x - 2}{(x - 1)(x + 3)}$

b. $y = \frac{x - 2}{(x - 2)(x + 3)}$

c. $y = \frac{x^2 - 1}{x + 1}$

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Examples

❸ **Finding Horizontal Asymptotes** Find the horizontal asymptote of $y = \frac{-4x + 3}{2x + 1}$.

Divide the numerator by the denominator as shown at the right.

$$
\begin{array}{r}
-2 \\
2x + 1 \overline{) \; -4x + 3} \\
-(-4x - \square) \\
\hline
5
\end{array}
$$

The function $y = \frac{-4x + 3}{2x + 1}$ can be written as $y = \frac{\square}{2x + 1} - \square$.

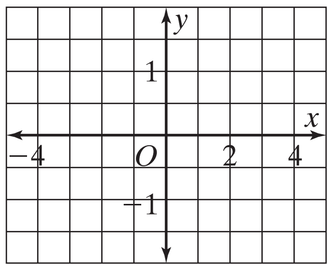
Its graph is a translation of $y = \frac{5}{2x + 1}$.

The horizontal asymptote is $y = \square$.

❹ **Sketching Graphs of Rational Functions** Sketch the graph $y = \frac{x + 1}{(x - 3)(x + 2)}$.

The degree of the denominator is greater than the degree of the numerator, so the x-axis is the ____________. When $x > 3$, y is positive. So as x increases, the graph approaches the y-axis from above. When $x < -2$, y is ________. So as x decreases, the graph approaches the y-axis from ________.

Since ____ is the zero of the numerator, the x-intercept is at ____. Since ____ and ____ are the zeros of the denominator, the vertical asymptotes are at $x =$ ____ and $x =$ ____.
Calculate the values of y for values of x near the asymptotes.
Plot those points and sketch the graph.



Quick Check

3. Find the horizontal asymptote of the graph of each rational function.

a. $y = \frac{-2x + 6}{x - 1}$

b. $y = \frac{2x^2 + 5}{x^2 + 1}$

4. Sketch the graph of $y = \frac{x + 3}{(x - 1)(x - 5)}$.