

Lesson 11-2

Arithmetic Sequences

Lesson Objective

▼ Identifying and generating arithmetic sequences

NAEP 2005 Strand: Algebra

Topic: Patterns, Relations, and Functions

Local Standards: _____

Vocabulary and Key Concepts

Arithmetic Sequence Formulas

Recursive Formula

$$a_1 = \text{a given value}, a_n = a_{n-1} + d$$

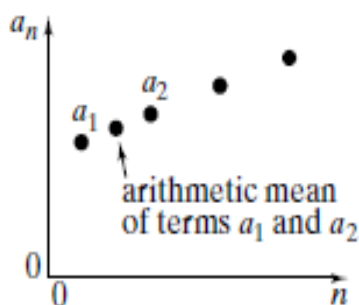
Explicit Formula

$$a_n = a_1 + (n - 1)d$$

In these formulas, a_n is the n th term, a_1 is the first term, n is the number of the term, and d is the common difference.

In an arithmetic sequence, **the difference between consecutive terms is constant.**

A common difference is **the difference between consecutive terms in an arithmetic sequence.**



The arithmetic mean of any two numbers is **their sum divided by two.**

Examples

1 Identifying an Arithmetic Sequence Is the given sequence arithmetic?

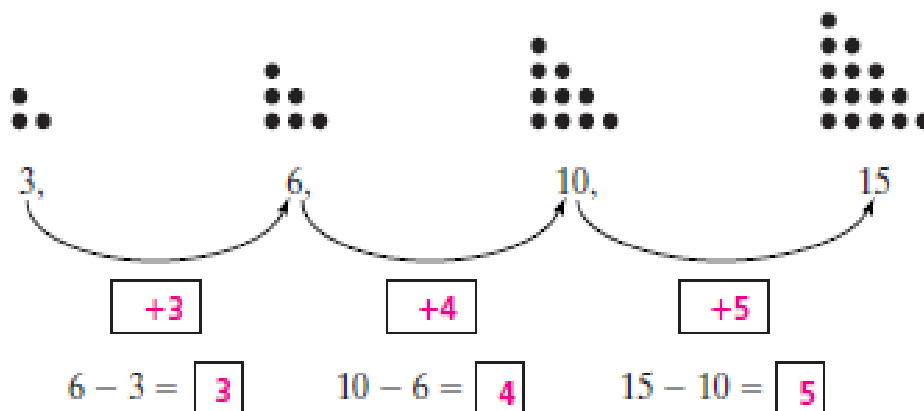
a. 7, 10, 13, 16, ...



$$10 - 7 = \boxed{3} \quad 13 - 10 = \boxed{3} \quad 16 - 13 = \boxed{3}$$

The common difference is $\boxed{+3}$. This $\boxed{\text{is}}$ an arithmetic sequence.

b. the sequence of dots in the triangles shown below



There is no common difference. This $\boxed{\text{is not}}$ an arithmetic sequence.

2 Saving Money Suppose you have already saved \$75 toward the purchase of a new CD player and speakers. You plan to save at least \$12 a week from money you earn at a part-time job. In all, what is the minimum amount you will have saved after 26 weeks?

Find the 27th term of the sequence 75, 87, 99, ...

$$a_n = a_1 + (n - 1)d$$

Use the explicit formula.

$$a_{27} = \boxed{75} + (\boxed{27} - \boxed{1})(\boxed{12}) \quad \text{Substitute } a_1 = \boxed{75}, n = \boxed{27}, \text{ and } d = \boxed{12}.$$

$$= 75 + (\boxed{26})(\boxed{12})$$

Subtract within parentheses.

$$= 75 + \boxed{312}$$

Multiply.

$$= \boxed{387}$$

Simplify.

After 26 weeks, you will have saved a minimum of \$ $\boxed{387}$.

- ③ **Using the Arithmetic Mean** Find the missing term of the arithmetic sequence 50, , 92.

$$\text{arithmetic mean} = \frac{\boxed{92} + \boxed{50}}{\boxed{2}} \quad \text{Write the average.}$$

$$= \frac{\boxed{142}}{\boxed{2}} \quad \text{Simplify the numerator.}$$

$$= \boxed{71} \quad \text{Divide.}$$

The missing term is 71 .

Quick Check

1. Is the given sequence arithmetic? If so, identify the common difference.

a. 2, 5, 7, 12, ...

no

b. 48, 45, 42, 39, ...

yes; -3

2. a. Refer to the formula in Example 2. Why was it necessary to find the value of the 27th term, not the 26th term?

The first term in the sequence, 75, was the amount of money you saved before you began setting aside money from your part-time job, so the first week of saving is actually the second term. The number of terms is one more than the number of weeks.

- b. Use the explicit formula to find the 25th term in the sequence 5, 11, 17, 23, 29, ...

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3. a. Find the missing term of the arithmetic sequence 24, , 57.

40.5

- b. Write an expression that shows the arithmetic mean of a_6 and a_7 .

$\frac{a_6 + a_7}{2}$